thm_2Estate__transformer_2EJOIN__MAP__JOIN (TML-bARsxdGz8SzXXbQ42hiiYg4jACgDD4W6)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 4 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 5 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Estate__transformer_2EUNIT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27b.(\lambda V1s$

Definition 11 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Epair\_2ESND : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Let $c\_2Epair\_2EFST : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Definition 12 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 13 *We define* c_2Estate__transformer_2EBIND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0g \in ((ty$

Definition 14 *We define* c_2Estate__transformer_2EMMAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ($

Definition 15 *We define* c_2Estate__transformer_2EJOIN *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0z \in ((ty\_2Epair\_2$

Definition 16 *We define* c_2Estate__transformer_2EEXT *to be* $\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27s : \iota.\lambda V0f \in (((ty$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}((\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27b) = (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EEXT}\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27b\ A\_27a)\ (\mathsf{c\_2Ecombin\_2EI}\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.nonempty\ A\_27c{\Rightarrow}(\forall V0f \in (((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})^{A\_27b}).((ap\ (\mathsf{c\_2Estate\_\_transformer\_2EEXT}\ A\_27b\ A\_27c\ A\_27a)\ V0f) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a}))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27c))\ (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EMMAP}\ A\_27a\ ((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})\ A\_27b)\ V0f)))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})^{A\_27b}). ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ A\_27c\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a}))\ (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EEXT}\ A\_27b\ A\_27c\ A\_27a)\ V0f))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27b)) = (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EEXT}\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27c\ A\_27a)\ (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EEXT}\ A\_27b\ A\_27c\ A\_27a)\ V0f)))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a}))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Estate\_\_transformer\_2EMMAP}\ A\_27a\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a}))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27b))) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ ((ty\_2Epair\_2Eprod\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a})\ A\_27a)^{A\_27a}))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ A\_27b))\ (\mathsf{c\_2Estate\_\_transformer\_2EJOIN}\ A\_27a\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a}))))$$