

thm_2Estate__transformer_2EMCOMP__ASSOC (TMZk6rMTcyRWyWDddC71CrRUDy96A5HqoHr)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (3)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))) (\lambda V1P \in 2.V1P)))$

Definition 4 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27b}))$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}). \lambda V2h \in (A_27a^{A_27b}). \lambda V3i \in (A_27b^{A_27c}). \lambda V4j \in (A_27c^{A_27a}). \lambda V5k \in (A_27a^{A_27b}). \lambda V6l \in (A_27b^{A_27c}). \lambda V7m \in (A_27c^{A_27a}). \lambda V8n \in (A_27a^{A_27b}). \lambda V9o \in (A_27b^{A_27c}). \lambda V10p \in (A_27c^{A_27a}). \lambda V11q \in (A_27a^{A_27b}). \lambda V12r \in (A_27b^{A_27c}). \lambda V13s \in (A_27c^{A_27a}). \lambda V14t \in (A_27a^{A_27b}). \lambda V15u \in (A_27b^{A_27c}). \lambda V16v \in (A_27c^{A_27a}). \lambda V17w \in (A_27a^{A_27b}). \lambda V18x \in (A_27b^{A_27c}). \lambda V19y \in (A_27c^{A_27a}). \lambda V20z \in (A_27a^{A_27b}). \lambda V21aa \in (A_27b^{A_27c}). \lambda V22ab \in (A_27c^{A_27a}). \lambda V23ba \in (A_27a^{A_27b}). \lambda V24bb \in (A_27b^{A_27c}). \lambda V25ca \in (A_27c^{A_27a}). \lambda V26cb \in (A_27a^{A_27b}). \lambda V27cb \in (A_27b^{A_27c}). \lambda V28ca \in (A_27c^{A_27a}). \lambda V29ca \in (A_27a^{A_27b}). \lambda V30ca \in (A_27b^{A_27c}). \lambda V31ca \in (A_27c^{A_27a}). \lambda V32ca \in (A_27a^{A_27b}). \lambda V33ca \in (A_27b^{A_27c}). \lambda V34ca \in (A_27c^{A_27a}). \lambda V35ca \in (A_27a^{A_27b}). \lambda V36ca \in (A_27b^{A_27c}). \lambda V37ca \in (A_27c^{A_27a}). \lambda V38ca \in (A_27a^{A_27b}). \lambda V39ca \in (A_27b^{A_27c}). \lambda V40ca \in (A_27c^{A_27a}). \lambda V41ca \in (A_27a^{A_27b}). \lambda V42ca \in (A_27b^{A_27c}). \lambda V43ca \in (A_27c^{A_27a}). \lambda V44ca \in (A_27a^{A_27b}). \lambda V45ca \in (A_27b^{A_27c}). \lambda V46ca \in (A_27c^{A_27a}). \lambda V47ca \in (A_27a^{A_27b}). \lambda V48ca \in (A_27b^{A_27c}). \lambda V49ca \in (A_27c^{A_27a}). \lambda V50ca \in (A_27a^{A_27b}). \lambda V51ca \in (A_27b^{A_27c}). \lambda V52ca \in (A_27c^{A_27a}). \lambda V53ca \in (A_27a^{A_27b}). \lambda V54ca \in (A_27b^{A_27c}). \lambda V55ca \in (A_27c^{A_27a}). \lambda V56ca \in (A_27a^{A_27b}). \lambda V57ca \in (A_27b^{A_27c}). \lambda V58ca \in (A_27c^{A_27a}). \lambda V59ca \in (A_27a^{A_27b}). \lambda V60ca \in (A_27b^{A_27c}). \lambda V61ca \in (A_27c^{A_27a}). \lambda V62ca \in (A_27a^{A_27b}). \lambda V63ca \in (A_27b^{A_27c}). \lambda V64ca \in (A_27c^{A_27a}). \lambda V65ca \in (A_27a^{A_27b}). \lambda V66ca \in (A_27b^{A_27c}). \lambda V67ca \in (A_27c^{A_27a}). \lambda V68ca \in (A_27a^{A_27b}). \lambda V69ca \in (A_27b^{A_27c}). \lambda V70ca \in (A_27c^{A_27a}). \lambda V71ca \in (A_27a^{A_27b}). \lambda V72ca \in (A_27b^{A_27c}). \lambda V73ca \in (A_27c^{A_27a}). \lambda V74ca \in (A_27a^{A_27b}). \lambda V75ca \in (A_27b^{A_27c}). \lambda V76ca \in (A_27c^{A_27a}). \lambda V77ca \in (A_27a^{A_27b}). \lambda V78ca \in (A_27b^{A_27c}). \lambda V79ca \in (A_27c^{A_27a}). \lambda V80ca \in (A_27a^{A_27b}). \lambda V81ca \in (A_27b^{A_27c}). \lambda V82ca \in (A_27c^{A_27a}). \lambda V83ca \in (A_27a^{A_27b}). \lambda V84ca \in (A_27b^{A_27c}). \lambda V85ca \in (A_27c^{A_27a}). \lambda V86ca \in (A_27a^{A_27b}). \lambda V87ca \in (A_27b^{A_27c}). \lambda V88ca \in (A_27c^{A_27a}). \lambda V89ca \in (A_27a^{A_27b}). \lambda V90ca \in (A_27b^{A_27c}). \lambda V91ca \in (A_27c^{A_27a}). \lambda V92ca \in (A_27a^{A_27b}). \lambda V93ca \in (A_27b^{A_27c}). \lambda V94ca \in (A_27c^{A_27a}). \lambda V95ca \in (A_27a^{A_27b}). \lambda V96ca \in (A_27b^{A_27c}). \lambda V97ca \in (A_27c^{A_27a}). \lambda V98ca \in (A_27a^{A_27b}). \lambda V99ca \in (A_27b^{A_27c}). \lambda V100ca \in (A_27c^{A_27a}).$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 9 We define $c_2Epair_2ECURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27c^{(ty_2Epair$

Definition 10 We define $c_2Estate_transformer_2EEXT$ to be $\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota.\lambda V0f \in (((ty$

Definition 11 We define $c_2Estate_transformer_2EMCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s :$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\ & (\forall V1g \in (A_27a^{A_27c}).(\forall V2h \in (A_27c^{A_27d}).((ap (\\ & ap (c_2Ecombin_2Eo A_27d A_27b A_27a) V0f) (ap (ap (c_2Ecombin_2Eo \\ & A_27d A_27a A_27c) V1g) V2h))) = (ap (ap (c_2Ecombin_2Eo A_27d A_27b \\ & A_27c) (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) V0f) V1g))) V2h)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f \in (A_27c^{(ty_2Epair_2Eprod A_27a A_27b)}). \\ & ((ap (c_2Epair_2EUNCURRY A_27a A_27b A_27c) (ap (c_2Epair_2ECURRY \\ & A_27a A_27b A_27c) V0f)) = V0f)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow (\forall V0g \in ((\\ & (ty_2Epair_2Eprod A_27c A_27b)^{A_27b})^{A_27d}).(\forall V1f \in ((\\ & (ty_2Epair_2Eprod A_27d A_27b)^{A_27b})^{A_27a}).((ap (ap (c_2Estate_transformer_2EMCOMP \\ & A_27a A_27d A_27c A_27b) V0g) V1f) = (ap (c_2Epair_2ECURRY A_27a \\ & A_27b (ty_2Epair_2Eprod A_27c A_27b)) (ap (ap (c_2Ecombin_2Eo \\ & (ty_2Epair_2Eprod A_27a A_27b) (ty_2Epair_2Eprod A_27c A_27b) \\ & (ty_2Epair_2Eprod A_27d A_27b)) (ap (c_2Epair_2EUNCURRY A_27d \\ & A_27b (ty_2Epair_2Eprod A_27c A_27b) V0g)) (ap (c_2Epair_2EUNCURRY \\ & A_27a A_27b (ty_2Epair_2Eprod A_27d A_27b)) V1f)))))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\ & A.27e \Rightarrow (\forall V0f \in (((ty_2Epair_2Eprod\ A.27c\ A.27b)^{A.27b})^{A.27d}). \\ & (\forall V1g \in (((ty_2Epair_2Eprod\ A.27d\ A.27b)^{A.27b})^{A.27e}). \\ & (\forall V2h \in (((ty_2Epair_2Eprod\ A.27e\ A.27b)^{A.27b})^{A.27a}). \\ & ((ap\ (ap\ (c.2Estate_transformer_2EMCOMP\ A.27a\ A.27d\ A.27c\ A.27b) \\ & V0f)\ (ap\ (ap\ (c.2Estate_transformer_2EMCOMP\ A.27a\ A.27e\ A.27d \\ & A.27b)\ V1g)\ V2h)) = (ap\ (ap\ (c.2Estate_transformer_2EMCOMP\ A.27a \\ & A.27e\ A.27c\ A.27b)\ (ap\ (ap\ (c.2Estate_transformer_2EMCOMP\ A.27e \\ & A.27d\ A.27c\ A.27b)\ V0f)\ V1g))\ V2h)))) \end{aligned}$$