

thm_2Estate__transformer_2EMCOMP__ID (TM- LvSHwueGowMK9d2HKGE3YLENxFdekNaYR)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define `c_2Ecombin_2EK` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define `c_2Ecombin_2ES` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 9 We define `c_2Ecombin_2EI` to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS__prod A_27a A_27b) (V0x V1y))$

Definition 11 We define `c_2Estate__transformer_2EUNIT` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27b.(\lambda V1s \in 2.V1s)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 13 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Definition 14 We define $c_2Epair_2ECURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27c^{(ty_2Epair$

Definition 15 We define $c_2Estate_transformer_2EEXT$ to be $\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota.\lambda V0f \in (((ty$

Definition 16 We define $c_2Estate_transformer_2EMCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s :$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\ A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\ A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).((ap\ (c_2Epair_2ECURRY \\ A_27a\ A_27b\ A_27c)\ (ap\ (c_2Epair_2EUNCURRY\ A_27a\ A_27b\ A_27c)\ V0f)) = \\ V0f)) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty\ A_{.27c} \Rightarrow (\forall V0f \in (A_{.27c}^{(ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})}), \\
& ((ap\ (c_2Epair_2EUNCURRY\ A_{.27a}\ A_{.27b}\ A_{.27c})\ (ap\ (c_2Epair_2ECURRY \\
& A_{.27a}\ A_{.27b}\ A_{.27c})\ V0f)) = V0f))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& (c_2Estate_transformer_2EUNIT\ A_{.27b}\ A_{.27a}) = (ap\ (c_2Epair_2ECURRY \\
& A_{.27a}\ A_{.27b}\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}))\ (c_2Ecombin_2EI\ (\\
& ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty\ A_{.27c} \Rightarrow \forall A_{.27d}.nonempty\ A_{.27d} \Rightarrow (\forall V0g \in ((\\
& (ty_2Epair_2Eprod\ A_{.27c}\ A_{.27b})^{A_{.27b}A_{.27d}}).(\forall V1f \in ((\\
& (ty_2Epair_2Eprod\ A_{.27d}\ A_{.27b})^{A_{.27b}A_{.27a}}).((ap\ (ap\ (c_2Estate_transformer_2EMCOMP \\
& A_{.27a}\ A_{.27d}\ A_{.27c}\ A_{.27b})\ V0g)\ V1f) = (ap\ (c_2Epair_2ECURRY\ A_{.27a} \\
& A_{.27b}\ (ty_2Epair_2Eprod\ A_{.27c}\ A_{.27b}))\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})\ (ty_2Epair_2Eprod\ A_{.27c}\ A_{.27b}) \\
& (ty_2Epair_2Eprod\ A_{.27d}\ A_{.27b}))\ (ap\ (c_2Epair_2EUNCURRY\ A_{.27d} \\
& A_{.27b}\ (ty_2Epair_2Eprod\ A_{.27c}\ A_{.27b}))\ V0g))\ (ap\ (c_2Epair_2EUNCURRY \\
& A_{.27a}\ A_{.27b}\ (ty_2Epair_2Eprod\ A_{.27d}\ A_{.27b}))\ V1f))))))
\end{aligned} \tag{12}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty\ A_{.27c} \Rightarrow \forall A_{.27d}.nonempty\ A_{.27d} \Rightarrow \forall A_{.27e}.nonempty \\
& A_{.27e} \Rightarrow \forall A_{.27f}.nonempty\ A_{.27f} \Rightarrow (\forall V0g \in (((ty_2Epair_2Eprod \\
& A_{.27c}\ A_{.27b})^{A_{.27b}A_{.27a}}).(\forall V1f \in (((ty_2Epair_2Eprod \\
& A_{.27f}\ A_{.27e})^{A_{.27e}A_{.27d}}).(((ap\ (ap\ (c_2Estate_transformer_2EMCOMP \\
& A_{.27a}\ A_{.27a}\ A_{.27c}\ A_{.27b})\ V0g)\ (c_2Estate_transformer_2EUNIT \\
& A_{.27b}\ A_{.27a})) = V0g) \wedge ((ap\ (ap\ (c_2Estate_transformer_2EMCOMP \\
& A_{.27d}\ A_{.27f}\ A_{.27f}\ A_{.27e})\ (c_2Estate_transformer_2EUNIT\ A_{.27e} \\
& A_{.27f}))\ V1f) = V1f))))))
\end{aligned}$$