

thm_2Estate__transformer_2EMMAP__JOIN
 (TMKYfm-
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 7 We define $c_2Estate_transformer_2EUNIT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27b.(\lambda V1s \in$

Definition 8 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 9 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})$

Definition 10 We define $c_2Estate_transformer_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ((ty$

Definition 11 We define $c_2Estate_transformer_2EMMAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A$

Definition 12 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 13 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 14 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 15 We define $c_2Estate_transformer_2EJOIN$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0z \in ((ty_2Epair_2E$

Definition 16 We define $c_2Estate_transformer_2EEXT$ to be $\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota.\lambda V0f \in (((ty$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}).((ap\ (c_2Estate_transformer_2EMMAP \\ A_27a\ A_27c\ A_27b)\ V0f) = (ap\ (c_2Estate_transformer_2EEXT\ A_27b \\ A_27c\ A_27a)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27b\ ((ty_2Epair_2Eprod \\ A_27c\ A_27a)^{A_27a})\ A_27c)\ (c_2Estate_transformer_2EUNIT\ A_27a \\ A_27c))\ V0f)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})^{A.27b}). \\
& ((ap\ (c_2Estate_transformer_2EEXT\ A.27b\ A.27c\ A.27a)\ V0f) = (\\
& ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Epair_2Eprod\ A.27b\ A.27a)^{A.27a}) \\
& ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod\ ((\\
& ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ A.27a)^{A.27a}))\ (c_2Estate_transformer_2EJOIN \\
& A.27a\ A.27c))\ (ap\ (c_2Estate_transformer_2EMMAP\ A.27a\ ((ty_2Epair_2Eprod \\
& A.27c\ A.27a)^{A.27a})\ A.27b)\ V0f))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})^{A.27b}). \\
& ((ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Epair_2Eprod\ ((ty_2Epair_2Eprod \\
& A.27b\ A.27a)^{A.27a})\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a}) \\
& ((ty_2Epair_2Eprod\ A.27b\ A.27a)^{A.27a}))\ (ap\ (c_2Estate_transformer_2EEXT \\
& A.27b\ A.27c\ A.27a)\ V0f))\ (c_2Estate_transformer_2EJOIN\ A.27a \\
& A.27b)) = (ap\ (c_2Estate_transformer_2EEXT\ ((ty_2Epair_2Eprod \\
& A.27b\ A.27a)^{A.27a})\ A.27c\ A.27a)\ (ap\ (c_2Estate_transformer_2EEXT \\
& A.27b\ A.27c\ A.27a)\ V0f))))
\end{aligned} \tag{11}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}).((ap\ (ap\ (c_2Ecombin_2Eo \\
& (ty_2Epair_2Eprod\ ((ty_2Epair_2Eprod\ A.27b\ A.27a)^{A.27a})\ A.27a)^{A.27a}) \\
& ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod\ A.27b \\
& A.27a)^{A.27a}))\ (ap\ (c_2Estate_transformer_2EMMAP\ A.27a\ A.27c \\
& A.27b)\ V0f))\ (c_2Estate_transformer_2EJOIN\ A.27a\ A.27b)) = (\\
& ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Epair_2Eprod\ ((ty_2Epair_2Eprod \\
& A.27b\ A.27a)^{A.27a})\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a}) \\
& ((ty_2Epair_2Eprod\ ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ A.27a)^{A.27a})) \\
& (c_2Estate_transformer_2EJOIN\ A.27a\ A.27c))\ (ap\ (c_2Estate_transformer_2EMMAP \\
& A.27a\ ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod \\
& A.27b\ A.27a)^{A.27a}))\ (ap\ (c_2Estate_transformer_2EMMAP\ A.27a \\
& A.27c\ A.27b)\ V0f))))
\end{aligned}$$