

thm_2Estate__transformer_2EUNIT__o__MCOMP (TMRsff29jMLE5xqDHjrNKTDNVqCVqdYxSSv)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (3)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 6 We define $c_2Estate_transformer_2EEXT$ to be $\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota.\lambda V0f \in (((ty_o$

Definition 7 We define $c_2Estate_transformer_2EMCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Definition 11 We define $c_2Estate_transformer_2EUNIT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27b. (\lambda V1s \in A_27a. (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1s))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\ & (\forall V1g \in (A_27a^{A_27c}). (\forall V2h \in (A_27c^{A_27d}). ((ap\ (\\ & ap\ (c_2Ecombin_2Eo\ A_27d\ A_27b\ A_27a)\ V0f)\ (ap\ (ap\ (c_2Ecombin_2Eo \\ & A_27d\ A_27a\ A_27c)\ V1g)\ V2h)) = (ap\ (ap\ (c_2Ecombin_2Eo\ A_27d\ A_27b \\ & A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g))\ V2h)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (((ty_2Epair_2Eprod\ A_27c\ A_27b)^{A_27b})^{A_27a}). \\ & ((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ((ty_2Epair_2Eprod\ A_27c\ A_27b)^{A_27b}) \\ & ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{A_27b})))\ (ap\ (c_2Estate_transformer_2EEXT \\ & A_27a\ A_27c\ A_27b)\ V0f))\ (c_2Estate_transformer_2EUNIT\ A_27b \\ & A_27a)) = V0f)) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0g \in (A_27c^{A_27d}). \\ & (\forall V1f \in (A_27d^{A_27a}). ((ap\ (ap\ (c_2Estate_transformer_2EMCOMP \\ & A_27a\ A_27d\ A_27c\ A_27b)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27d\ ((ty_2Epair_2Eprod \\ & A_27c\ A_27b)^{A_27b})\ A_27c)\ (c_2Estate_transformer_2EUNIT\ A_27b \\ & A_27c))\ V0g))\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ((ty_2Epair_2Eprod \\ & A_27d\ A_27b)^{A_27b})\ A_27d)\ (c_2Estate_transformer_2EUNIT\ A_27b \\ & A_27d))\ V1f)) = (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ((ty_2Epair_2Eprod \\ & A_27c\ A_27b)^{A_27b})\ A_27c)\ (c_2Estate_transformer_2EUNIT\ A_27b \\ & A_27c))\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27d)\ V0g)\ V1f)))))) \end{aligned}$$