

thm_2Estate_transformer_2EmapM_nil
(TMdARGhgKkMRmShGkaTiTXPHDXCJKbt-
dCDU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ECONS A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})^{A.27a}) \quad (2)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap (ap (c_2Emin_2E_3D (2^{A.27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Elist_2EMAP A.27a A.27b \in (((ty_2Elist_2Elist A.27b)^{(ty_2Elist_2Elist A.27a)})^{(A.27b^{A.27a})}) \quad (3)$$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A.27c}).\lambda V1g$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Definition 8 We define $c_2Estate_transformer_2EUNIT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27b.(\lambda V1s \in$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (7)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (8)$$

Definition 9 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})$

Definition 10 We define $c_2Estate_transformer_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0g \in ((ty$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{(A_27b^{A_27b})^{A_27a}}) \quad (9)$$

Definition 11 We define $c_2Estate_transformer_2Esequence$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (ap\ (c_2Elist_2$

Definition 12 We define $c_2Estate_transformer_2EmapM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (($

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\
& (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27c\ A.27b\ A.27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b) \\
& V0f)\ (c.2Elist_2ENIL\ A.27a)) = (c.2Elist_2ENIL\ A.27b))) \wedge (\forall V1f \in \\
& (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c.2Elist.2EMAP\ A.27a\ A.27b)\ V1f)\ V3t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (ap\ (c.2Estate_transformer_2Esequence\ A.27a\ A.27b)\ (c.2Elist_2ENIL \\
& ((ty.2Epair.2Eprod\ A.27b\ A.27a)^{A.27a}))) = (ap\ (c.2Estate_transformer.2EUNIT \\
& A.27a\ (ty.2Elist.2Elist\ A.27b))\ (c.2Elist.2ENIL\ A.27b)))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (((ty.2Epair.2Eprod\ A.27b\ A.27a)^{A.27a})^{A.27c}). \\
& ((ap\ (ap\ (c.2Estate_transformer.2EmapM\ A.27c\ A.27a\ A.27b)\ V0f) \\
& (c.2Elist.2ENIL\ A.27c)) = (ap\ (c.2Estate_transformer.2EUNIT \\
& A.27a\ (ty.2Elist.2Elist\ A.27b))\ (c.2Elist.2ENIL\ A.27b))))
\end{aligned}$$