

thm\_2Estate\_\_transformer\_2Esequence\_\_nil  
(TMYnkhE3J6TTAWekR6GAHmsWRSrM8bbaTcR)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (2)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (3)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

**Definition 7** We define  $c\_2Estate\_transformer\_2EUNIT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27b.(\lambda V1s \in$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (6)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (7)$$

**Definition 8** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

**Definition 9** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in$

**Definition 10** We define  $c\_2Estate\_transformer\_2EBIND$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0g \in ((ty$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDR \\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})_{A\_27b})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (8)$$

**Definition 11** We define  $c\_2Estate\_transformer\_2Esequence$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (ap (c\_2Elist\_2$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27b.((ap ( \\ ap (ap (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e) (c\_2Elist\_2ENIL \\ A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V3e \in \\ A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a). \\ ((ap (ap (ap (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V2f)\ V3e) (ap (ap (c\_2Elist\_2ECONS \\ A\_27a)\ V4x)\ V5l)) = (ap (ap V2f\ V4x) (ap (ap (ap (c\_2Elist\_2EFOLDR \\ A\_27a\ A\_27b)\ V2f)\ V3e)\ V5l)))))))))) \end{aligned} \quad (11)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (ap (c\_2Estate\_transformer\_2Esequence\ A\_27a\ A\_27b) (c\_2Elist\_2ENIL \\ ((ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)^{A\_27a}))) = (ap (c\_2Estate\_transformer\_2EUNIT \\ A\_27a\ (ty\_2Elist\_2Elist\ A\_27b)) (c\_2Elist\_2ENIL\ A\_27b))) \end{aligned}$$