

thm_2Estring_2EDEST__STRING__LEMS (TMWHSTvRKEBq1JssgVLrnZ1g5ajfstMe7kH)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 8 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (ty_2Epair_2Eprod A_27a A_27b))$

Let $ty_2Eone_2Eone : \iota \Rightarrow \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (7)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (9)$$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 16 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0e)$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty_2Estring_2Echar \quad (11)$$

Let $c_2Estring_2EDEST_STRING : \iota$ be given. Assume the following.

$$c_2Estring_2EDEST_STRING \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist\ ty_2Estring_2Echar)))^{(ty_2Elist_2Elist\ ty_2Estring_2Echar)}) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (16)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((V0l = (c_2Elist_2ENIL\ A_27a)) \vee (\exists V1h \in A_27a. (\exists V2t \in (ty_2Elist_2Elist\ A_27a). (V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t)))))) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a. (\forall V1a1 \in \\ & (ty_2Elist_2Elist\ A_27a). (\forall V2a0_27 \in A_27a. (\forall V3a1_27 \in \\ & (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\ & V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = \\ & V2a0_27) \wedge (V1a1 = V3a1_27)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\ & A_27a). (\forall V1a0 \in A_27a. (\neg((c_2Elist_2ENIL\ A_27a) = (ap\ (\\ & ap\ (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\ & A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (((ap\ c_2Estring_2EDEST_STRING\ (c_2Elist_2ENIL\ ty_2Estring_2Echar)) = \\ & (c_2Eoption_2ENONE\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist \\ & ty_2Estring_2Echar)))) \wedge (\forall V0c \in ty_2Estring_2Echar. (\\ & \forall V1rst \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). ((ap\ c_2Estring_2EDEST_STRING \\ & (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Estring_2Echar)\ V0c)\ V1rst) = (ap \\ & (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist \\ & ty_2Estring_2Echar)))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Estring_2Echar \\ & (ty_2Elist_2Elist\ ty_2Estring_2Echar)\ V0c)\ V1rst)))))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} & (\forall V0c \in ty_2Estring_2Echar. (\forall V1t \in (ty_2Elist_2Elist \\ & ty_2Estring_2Echar). (\forall V2s \in (ty_2Elist_2Elist ty_2Estring_2Echar). \\ & (((ap c_2Estring_2EDEST_STRING V2s) = (c_2Eoption_2ENONE (\\ & ty_2Epair_2Eprod ty_2Estring_2Echar (ty_2Elist_2Elist ty_2Estring_2Echar)))) \Leftrightarrow \\ & (V2s = (c_2Elist_2ENIL ty_2Estring_2Echar))) \wedge (((ap c_2Estring_2EDEST_STRING \\ & V2s) = (ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod ty_2Estring_2Echar \\ & (ty_2Elist_2Elist ty_2Estring_2Echar))) (ap (ap (c_2Epair_2E_2C \\ & ty_2Estring_2Echar (ty_2Elist_2Elist ty_2Estring_2Echar)) \\ & V0c) V1t))) \Leftrightarrow (V2s = (ap (ap (c_2Elist_2ECONS ty_2Estring_2Echar \\ & V0c) V1t)))))) \end{aligned}$$