thm_2Estring_2EEXTRACT__def (TMNEoZCRVEXRBzvf4MS1ETgCoiURLMrg6c8)

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Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (2)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (3)$$

Definition 6 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

Definition 7 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 13 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 14 *We define* c_2Erelation_2EEMPTY__REL *to be* $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27a.$c_2E

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{6}$$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{7}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{8}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{11}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{12}$$

Definition 17 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $ty\_2Estring\_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{13}$$

Let $c\_2Estring\_2ESUBSTRING : \iota$ be given. Assume the following.

$$c\_2Estring\_2ESUBSTRING \in ((ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)^{(ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist} \tag{14}$$

Definition 18 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 19 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 20 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a:\iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{15}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{16}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{17}$$

Definition 21 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0p \in (ty\_2Epair\_$

Definition 22 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 23 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a:\iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Definition 24 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a:\iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 25 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 26 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 27 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 28 *We define* c_2Estring_2EEXTRACT *to be* $(ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ (ty\_2Epair\_2Epro$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V1R)) \Rightarrow (\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))))) \tag{22}$$

Theorem 1

$$(\forall V0s \in (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).(\forall V1i \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Estring\_2EEXTRACT}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum)))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ (ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum))\ V1i)\ (\mathsf{c\_2Eoption\_2ENONE}\ ty\_2Enum\_2Enum)))) = (ap\ c\_2Estring\_2ESUBSTRING\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1i)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Elist\_2ELENGTH\ ty\_2Estring\_2Echar)\ V0s))\ V1i)))))\wedge((ap\ \mathsf{c\_2Estring\_2EEXTRACT}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum)))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ (ty\_2Eoption\_2Eoption\ ty\_2Enum\_2Enum))\ V1i)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ ty\_2Enum\_2Enum)\ V2n)))) = (ap\ c\_2Estring\_2ESUBSTRING\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1i)\ V2n))))))))$$