

thm_2Estring_2EisPREFIX_DEF (TMRwXFX- CdD3nvJrGCAw2gc6KymrqkTXbG1g)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2Elist_CASE A_27a A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27a}}))_{A_27b}(ty_2Elist_2Elist A_27a)) \quad (2)$$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EisPREFIX A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})_{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (5)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (7)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (8)$$

Definition 10 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Eprod\ A_27a\ A_27b)$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(c_2Ebool_2E_21\ 2)\ t2))\ t1))\ t1$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b)\ x\ y)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (10)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (11)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (12)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (13)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (14)$$

Definition 15 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Definition 17 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (15)$$

Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty ty_2Estring_2Echar \quad (16)$$

Let $c_2Estring_2EDEST_STRING : \iota$ be given. Assume the following.

$$c_2Estring_2EDEST_STRING \in ((ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Estring_2Echar (ty_2Elist_2Elist ty_2Estring_2Echar)))^{(ty_2Elist_2Elist ty_2Estring_2Echar)}) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(V0l = (c_2Elist_2ENIL\ A_27a)) \vee (\exists V1h \in A_27a.(\\ & \exists V2t \in (ty_2Elist_2Elist\ A_27a).(V0l = (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V1h)\ V2t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL \\ & A_27a))\ V0l)) \Leftrightarrow True)) \wedge (\forall V1h \in A_27a.(\forall V2t \in (ty_2Elist_2Elist \\ & A_27a).(\forall V3l \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t))\ V3l)) \Leftrightarrow (p\ (ap\ (\\ & ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ 2)\ V3l)\ c_2Ebool_2EF)\ (\lambda V4h_27 \in \\ & A_27a.(\lambda V5t_27 \in (ty_2Elist_2Elist\ A_27a).(ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1h)\ V4h_27))\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\ & A_27a)\ V2t)\ V5t_27)))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1h \in A_27a.(\forall V2t \in (ty_2Elist_2Elist\ A_27a). \\ & (\forall V3h1 \in A_27a.(\forall V4t1 \in (ty_2Elist_2Elist\ A_27a). \\ & (\forall V5h2 \in A_27a.(\forall V6t2 \in (ty_2Elist_2Elist\ A_27a). \\ & (((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) \\ & V0l)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t))\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow False) \wedge \\ & ((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V3h1)\ V4t1))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5h2)\ V6t2))) \Leftrightarrow \\ & ((V3h1 = V5h2) \wedge (p\ (ap\ (ap\ (c_2Elist_2EisPREFIX\ A_27a)\ V4t1)\ V6t2)))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\ & (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b.(\forall V1y \in A_27c.(\forall V2f \in \\ & ((A_27a^{A_27c})^{A_27b}).((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\ & A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Estring_2EDEST_STRING\ (c_2Elist_2ENIL\ ty_2Estring_2Echar)) = \\
& \quad (c_2Eoption_2ENONE\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist \\
& \quad \quad ty_2Estring_2Echar)))) \wedge (\forall V0c \in ty_2Estring_2Echar. (\\
\forall V1rst \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). ((ap\ c_2Estring_2EDEST_STRING \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Estring_2Echar)\ V0c)\ V1rst)) = (ap \\
& \quad (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist \\
& \quad \quad ty_2Estring_2Echar)))\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Estring_2Echar \\
& \quad \quad \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))\ V0c)\ V1rst)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s1 \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (\forall V1s2 \in \\
& \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar). ((p\ (ap\ (ap\ (c_2Elist_2EisPREFIX \\
& \quad \quad ty_2Estring_2Echar)\ V0s1)\ V1s2)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Epair_2Epair_CASE \\
& \quad \quad \quad 2\ (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar \\
& \quad \quad \quad \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar)))\ (ty_2Eoption_2Eoption \\
& \quad \quad \quad \quad (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist\ ty_2Estring_2Echar)))) \\
& \quad \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod \\
& \quad \quad \quad \quad ty_2Estring_2Echar\ (ty_2Elist_2Elist\ ty_2Estring_2Echar))) \\
& \quad \quad \quad \quad (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar \\
& \quad \quad \quad \quad \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))))))\ (ap\ c_2Estring_2EDEST_STRING \\
V0s1))\ (ap\ c_2Estring_2EDEST_STRING\ V1s2)))\ (\lambda V2v \in (ty_2Eoption_2Eoption \\
& \quad (ty_2Epair_2Eprod\ ty_2Estring_2Echar\ (ty_2Elist_2Elist\ ty_2Estring_2Echar))). \\
& \quad (\lambda V3v1 \in (ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ ty_2Estring_2Echar \\
& \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))). (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad \quad \quad 2)\ V2v)\ c_2Ebool_2ET)\ (\lambda V4v2 \in (ty_2Epair_2Eprod\ ty_2Estring_2Echar \\
& \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))). (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad \quad \quad 2)\ V3v1)\ c_2Ebool_2EF)\ (\lambda V5v3 \in (ty_2Epair_2Eprod\ ty_2Estring_2Echar \\
& \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))). (ap\ (ap\ (c_2Epair_2Epair_CASE \\
& \quad \quad \quad 2\ ty_2Estring_2Echar\ (ty_2Elist_2Elist\ ty_2Estring_2Echar)) \\
V5v3)\ (\lambda V6c2 \in ty_2Estring_2Echar. (\lambda V7t2 \in (ty_2Elist_2Elist \\
& \quad ty_2Estring_2Echar). (ap\ (ap\ (c_2Epair_2Epair_CASE\ 2\ ty_2Estring_2Echar \\
& \quad (ty_2Elist_2Elist\ ty_2Estring_2Echar))\ V4v2)\ (\lambda V8c1 \in ty_2Estring_2Echar. \\
& \quad (\lambda V9t1 \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad \quad (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Estring_2Echar)\ V8c1)\ V6c2))\ (ap\ (ap \\
& \quad \quad \quad \quad (c_2Elist_2EisPREFIX\ ty_2Estring_2Echar)\ V9t1)\ V7t2))))))))))))))
\end{aligned}$$