

thm\_2Estring\_2EisPREFIX\_STRCAT  
(TMTvEPAnPAHjLco8fPcbcqGyrkh3tP8taj7)

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Let  $ty\_2Estring\_2Echar : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P (ap (c\_2Emin\_2E40\ A\_27a\ V0P))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)(ty\_2Elist\_2Elist\ A\_27a))(ty\_2Elist\_2Elist\ A\_27a)) \tag{3}$$

Let  $c\_2Elist\_2Elist\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b \in (((A\_27b((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}))_{A\_27b}(ty\_2Elist\_2Elist\ A\_27a)) \tag{4}$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)(ty\_2Elist\_2Elist\ A\_27a))_{A\_27a} \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2EET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (6)$$

Let  $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (7)$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V2t \in 2. (ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V0t1)\ V2t))))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V2t \in 2. (ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V0t1)\ V2t))))$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_5C\_2F))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A\_27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A\_27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ (2^{A.27a}).((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \vee (\exists V4x \in A.27a.(p \ ( \\ ap \ V1Q \ V4x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((p \ V0P) \vee (\exists V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0a \in A.27a.(\exists V1x \in \\ A.27a.(V1x = V0a))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in \\ & A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).( \\ & \quad \forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad (\forall V0v \in A\_27b.(\forall V1f \in ((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). \\ & ((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b)\ (c\_2Elist\_2ENIL \\ & \quad A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A\_27a.(\forall V3a1 \in \\ & \quad (ty\_2Elist\_2Elist\ A\_27a).(\forall V4v \in A\_27b.(\forall V5f \in ( \\ & \quad (A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE \\ & \quad A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\ & \quad (ap\ (ap\ V5f\ V2a0)\ V3a1)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) \\ & \quad V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in \\ & (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & \quad A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap \\ & \quad (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a) \\ & \quad V1l1)\ V2l2)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ & ((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & \quad A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\ & \quad c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & \quad A\_27a).(p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\ & \quad A\_27a).((V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.( \\ & \quad \exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \\ & \quad \quad A\_27a)\ V1h)\ V2t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a. (\forall V1a1 \in \\ (ty\_2Elist\_2Elist\ A\_27a). (\forall V2a0\_27 \in A\_27a. (\forall V3a1\_27 \in \\ (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0) \\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = \\ V2a0\_27) \wedge (V1a1 = V3a1\_27))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist \\ A\_27a). (\forall V1a0 \in A\_27a. (\neg((c\_2Elist\_2ENIL\ A\_27a) = (ap\ ( \\ ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a0)\ V0a1)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V2l3 \in \\ (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a) \\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))\ V2l3)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist \\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX\ A\_27a)\ (c\_2Elist\_2ENIL \\ A\_27a))\ V0l)) \Leftrightarrow True)) \wedge (\forall V1h \in A\_27a. (\forall V2t \in (ty\_2Elist\_2Elist \\ A\_27a). (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1h)\ V2t))\ V3l)) \Leftrightarrow (p\ (ap\ ( \\ ap\ (ap\ (c\_2Elist\_2Elist\_CASE\ A\_27a\ 2)\ V3l)\ c\_2Ebool\_2EF)\ (\lambda V4h\_27 \in \\ A\_27a. (\lambda V5t\_27 \in (ty\_2Elist\_2Elist\ A\_27a). (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C \\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ A\_27a)\ V1h)\ V4h\_27))\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ A\_27a)\ V2t)\ V5t\_27)))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (43)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (51)$$

### Theorem 1

$$\begin{aligned} & (\forall V0s1 \in (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).(\forall V1s2 \in \\ & (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).((p (ap (ap (c\_2Elist\_2EisPREFIX \\ & ty\_2Estring\_2Echar) V0s1) V1s2)) \Leftrightarrow (\exists V2s3 \in (ty\_2Elist\_2Elist \\ & ty\_2Estring\_2Echar).(V1s2 = (ap (ap (c\_2Elist\_2EAPPEND ty\_2Estring\_2Echar) \\ & V0s1) V2s3)))))) \end{aligned}$$