thm_2Estring__num_2En2s__def (TMZxbfQ1RSJqwzxEHmnPRHyWXo2TPLpHKn6)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27})$

Definition 4 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (1)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (2)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \qquad (3)$$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (6)$$

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Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \land p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (7)$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Definition 20 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \qquad (11)$$

Definition 21 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1l$

Let $ty\_2Estring\_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{12}$$

Let $c\_2Estring\_2ECHR : \iota$ be given. Assume the following.

$$c\_2Estring\_2ECHR \in (ty\_2Estring\_2Echar^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{14}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{15}$$

Definition 22 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\\ A\_27a) \tag{16}$$

Definition 23 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 24 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27}$

Definition 25 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES$\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 26 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 27 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ $c_2Earithmetic

Definition 28 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ $c_2Earithmetic

Definition 29 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 30 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Definition 31 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 32 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 33 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 34 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 35 *We define* c_2Estring__num_2En2s *to be* $(ap$ $(ap$ (c_2Erelation_2EWFREC ty_2Enum_2Enum

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\\
V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))$$
$$(20)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V1n)\ V0m))))$$
$$(21)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n)))))$$
$$(22)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
\mathsf{c\_2Enum\_2E0})\ V0n)))$$
$$(23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V1n)\ V0m)))))$$
$$(24)$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge ($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n)))))))))))$$

(25)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ ($$
$$ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$

(26)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap$$
$$\textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V2p))))))$$

(27)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V0n)\ V1m))\ V0n))))$$

(28)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(V0m = V1n) \Leftrightarrow ((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \wedge (p\ ($$
$$ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))))$$

(29)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))$$

(30)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V0m))\ V1n)) \vee (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n))\ V0m))))))$$

$$(31)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\ V0n)))$$

$$(32)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ ($$
$$ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V2p)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0m)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2B}\ V1n)\ V2p))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\mathsf{c\_2Enum\_2E0})\ V2p)))))))$$

$$(33)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1d \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))$$
$$V1d))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EDIV$$
$$V0n)\ V1d))\ V0n)))))$$

$$(34)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Enum\_2Enum.($$
$$\forall V2y \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1x)\ V2y))) \Rightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2EDIV$$
$$V1x)\ V0n))\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V2y)\ V0n)))))))$$

$$(35)$$

Assume the following.
$$True \tag{36}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p$$
$$V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{40}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\wedge \\ ((p\ V1t2)\wedge(p\ V2t3))){\Leftrightarrow}(((p\ V0t1)\wedge(p\ V1t2))\wedge(p\ V2t3)))))) \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\wedge True{\Leftrightarrow} \\ (p\ V0t))\wedge(((False\wedge(p\ V0t)){\Leftrightarrow}False)\wedge((((p\ V0t)\wedge False){\Leftrightarrow}False)\wedge \\ (((p\ V0t)\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t)))))))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t)\vee True){\Leftrightarrow}True)\wedge \\ (((False\vee(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\vee False){\Leftrightarrow}(p\ V0t))\wedge(((p\ V0t)\vee \\ (p\ V0t)){\Leftrightarrow}(p\ V0t)))))))$$

$$\tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow} \\ True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((\\ (p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))))) \tag{44}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t)))\wedge(((\neg True){\Leftrightarrow}False)\wedge \\ ((\neg False){\Leftrightarrow}True))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow} \\ True)) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow} \\ (p\ V0t))\wedge(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t)))\wedge(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(\\ p\ V0t)))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2))))$$

$$(49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ($$
$$(p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee$$
$$(p\ V0A))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg($$
$$p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))))$$

$$(52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee$$
$$(p\ V1B))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow$$
$$((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (55)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p$$
$$V0t1) \wedge (p\ V1t2)) \vee ((\neg(p\ V0t1)) \wedge (\neg(p\ V1t2)))))))) \quad (56)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a.$$
$$(\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ V1b)\ (ap\ V0f$$
$$V2x))\ (ap\ V0f\ V3y)))))))$$

$$(57)$$

8

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).(\forall V2M \in$$
$$A\_27a.(\forall V3N \in A\_27a.(((V2M = V3N) \wedge (\forall V4x \in A\_27a.($$
$$(V4x = V3N) \Rightarrow ((ap\ V0f\ V4x) = (ap\ V1g\ V4x))))) \Rightarrow ((ap\ (ap\ (\text{c\_2Ebool\_2ELET}$$
$$A\_27a\ A\_27b)\ V0f)\ V2M) = (ap\ (ap\ (\text{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V1g)$$
$$V3N)))))))$$

$$(58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\text{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\text{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27))))))))))$$

$$(59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in$$
$$A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\qquad (60)$$
$$ap\ V0f\ V1v)))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\text{c\_2Ecombin\_2EI}$$
$$A\_27a)\ V0x) = V0x)) \qquad (61)$$

9

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\textsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \textsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V4n) = \textsf{c\_2Enum\_2E0}))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\textsf{c\_2Enum\_2E0})\ V8n) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \textsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \textsf{c\_2Enum\_2E0}) =$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \textsf{c\_2Enum\_2ESUC}$
$\textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$\textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ \textsf{c\_2Enum\_2E0}) =$
$\textsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \textsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\textsf{c\_2Enum\_2E0} = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n))\Leftrightarrow(V20n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}$
$\textsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\textsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))))))))))))$$

$$(63)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))$$

$$(64)$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}))\quad(65)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0R))\Rightarrow(\forall V1P \in (2^{A\_27a}).$$
$$((\forall V2x \in A\_27a.((\forall V3y \in A\_27a.((p\ (ap\ (ap\ V0R\ V3y)\ V2x))\Rightarrow$$
$$(p\ (ap\ V1P\ V3y))))\Rightarrow(p\ (ap\ V1P\ V2x))))\Rightarrow(\forall V4x \in A\_27a.(p\ (ap$$
$$V1P\ V4x)))))))$$

$$(66)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0f \in (A\_27b^{A\_27a}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}).(\forall V2y \in$$
$$A\_27a.(\forall V3z \in A\_27a.((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap$$
$$(ap\ \textsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V0f)\ V1R)\ V3z)\ V2y) =$$
$$(ap\ V0f\ V2y)))))))$$

$$(67)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (\textsf{c\_2Erelation\_2EWFREC}$$
$$A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (\textsf{c\_2Erelation\_2EWF}\ A\_27a)\ V1R)) \Rightarrow$$
$$(\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ \textsf{c\_2Erelation\_2ERESTRICT}$$
$$A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x))))))))$$

$$(68)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg($$
$$p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee$$
$$((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))))$$

$$(74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee$$
$$(\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))$$

$$(75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge$$
$$((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee$$
$$(p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))) \tag{77}$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Estring\_\_num\_2En2s}\ V0n) =$$
$$(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar))$$
$$(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V0n)\ \mathsf{c\_2Enum\_2E0}))\ (c\_2Elist\_2ENIL$$
$$ty\_2Estring\_2Echar))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ ty\_2Enum\_2Enum$$
$$(ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar))\ (\lambda V1r0 \in ty\_2Enum\_2Enum.$$
$$(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ ty\_2Enum\_2Enum\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar))$$
$$(\lambda V2r \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Elist\_2Elist$$
$$ty\_2Estring\_2Echar)\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar))$$
$$(\lambda V3s0 \in (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).(ap\ (ap\ (c\_2Elist\_2ECONS$$
$$ty\_2Estring\_2Echar)\ (ap\ \mathsf{c\_2Estring\_2ECHR}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V2r)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))))) \ V3s0)))\ (ap\ \mathsf{c\_2Estring\_\_num\_2En2s}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)$$
$$V2r))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))))))$$
$$(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}$$
$$ty\_2Enum\_2Enum)\ V1r0)\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))))$$
$$V1r0))))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))))))))))$$