

# thm\_2Estring\_\_num\_2En2s\_\_s2n (TMbDn- mfFL4Bq2NAQtLo45BFaAUD1GVX7mQN)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ V0P))$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \tag{1}$$

Let `c_2Earithmetic_2E_2A` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let `c_2Earithmetic_2E_2B` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

**Definition 4** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x))) \ (\lambda V1x \in 2. V1x)$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ V0P))$

**Definition 6** We define `c_2Earithmetic_2EMODEQ` to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1m1 \in ty\_2Enum.$

**Definition 7** We define `c_2Ebool_2EBOUNDED` to be  $(\lambda V0v \in 2. c_2Ebool_2ET).$

**Definition 8** We define `c_2Ecombin_2EC` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

**Definition 9** We define `c_2Ecombin_2Eo` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27a^{A_27c}).$

**Definition 10** We define `c_2Ecombin_2ES` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

**Definition 11** We define  $c\_E\_marker\_E\_Abbrev$  to be  $\lambda V0x \in 2.V0x$ .

Let  $c\_E\_arithmic\_E\_EVEN : \iota$  be given. Assume the following.

$$c\_E\_arithmic\_E\_EVEN \in (2^{ty\_E\_Enum\_E\_Enum}) \quad (4)$$

Let  $c\_E\_arithmic\_E\_ODD : \iota$  be given. Assume the following.

$$c\_E\_arithmic\_E\_ODD \in (2^{ty\_E\_Enum\_E\_Enum}) \quad (5)$$

**Definition 12** We define  $c\_E\_bool\_E\_EF$  to be  $(ap (c\_E\_bool\_E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_E\_min\_E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_E\_bool\_E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_E\_min\_E\_3D\_3D\_3E V0t) c\_E\_bool\_E\_21\ 2))$

**Definition 15** We define  $c\_E\_bool\_E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_E\_bool\_E\_21\ 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_E\_Enum\_E\_EREP\_num : \iota$  be given. Assume the following.

$$c\_E\_Enum\_E\_EREP\_num \in (\omega^{ty\_E\_Enum\_E\_Enum}) \quad (6)$$

Let  $c\_E\_Enum\_E\_ESUC\_REP : \iota$  be given. Assume the following.

$$c\_E\_Enum\_E\_ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

Let  $c\_E\_Enum\_E\_EABS\_num : \iota$  be given. Assume the following.

$$c\_E\_Enum\_E\_EABS\_num \in (ty\_E\_Enum\_E\_Enum^{\omega}) \quad (8)$$

**Definition 16** We define  $c\_E\_Enum\_E\_ESUC$  to be  $\lambda V0m \in ty\_E\_Enum\_E\_Enum.(ap c\_E\_Enum\_E\_EABS\_num)$

**Definition 17** We define  $c\_E\_prim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_E\_Enum\_E\_Enum.\lambda V1n \in ty\_E\_Enum\_E\_Enum$

**Definition 18** We define  $c\_E\_arithmic\_E\_3E$  to be  $\lambda V0m \in ty\_E\_Enum\_E\_Enum.\lambda V1n \in ty\_E\_Enum\_E\_Enum$

**Definition 19** We define  $c\_E\_bool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_E\_bool\_E\_21\ 2) (\lambda V2t \in 2.V2t))))$

**Definition 20** We define  $c\_E\_arithmic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_E\_Enum\_E\_Enum.\lambda V1n \in ty\_E\_Enum\_E\_Enum$

Let  $c\_E\_arithmic\_E\_EEXP : \iota$  be given. Assume the following.

$$c\_E\_arithmic\_E\_EEXP \in ((ty\_E\_Enum\_E\_Enum^{ty\_E\_Enum\_E\_Enum})^{ty\_E\_Enum\_E\_Enum}) \quad (9)$$

**Definition 21** We define  $c\_E\_numeral\_E\_EiSUC$  to be  $\lambda V0n \in ty\_E\_Enum\_E\_Enum.(ap c\_E\_Enum\_E\_ESUC (ap$

**Definition 22** We define  $c\_E\_arithmic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_E\_Enum\_E\_Enum.\lambda V1n \in ty\_E\_Enum\_E\_Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{10}$$

**Definition 23** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 24** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 25** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c\_2Enumeral\_2EiSUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \tag{11}$$

**Definition 26** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E$

**Definition 27** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{12}$$

**Definition 28** We define  $c\_2ERelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1$

Let  $ty\_2Estring\_2Echar : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{13}$$

Let  $c\_2Estring\_2ECHR : \iota$  be given. Assume the following.

$$c\_2Estring\_2ECHR \in (ty\_2Estring\_2Echar^{ty\_2Enum\_2Enum}) \tag{14}$$

**Definition 29** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27$

**Definition 30** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x)$

**Definition 31** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

**Definition 32** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E21$

**Definition 33** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 34** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 35** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 36** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 37** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 38** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 39** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 40** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (18)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (19)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (20)$$

**Definition 41** We define  $c\_2Estring\_num\_2En2s$  to be  $(ap (ap (c\_2Erelation\_2EWFREC ty\_2Enum\_2Enum$

Let  $c\_2Estring\_2EORD : \iota$  be given. Assume the following.

$$c\_2Estring\_2EORD \in (ty\_2Enum\_2Enum^{ty\_2Estring\_2Echar}) \quad (21)$$

Let  $c\_2Estring\_num\_2Es2n : \iota$  be given. Assume the following.

$$c\_2Estring\_num\_2Es2n \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist ty\_2Estring\_2Echar)}) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
& \quad ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& \quad V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V0n))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1n) V0m)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V0m) c\_2Enum\_2E0) = V0m))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m)) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A V1n) V0m)))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap (ap c\_2Earithmetic\_2E\_2A V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2A (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V2p)))))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0)))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V1n)) \Rightarrow ((V0m = (ap c\_2Eprim\_rec\_2EPRE V1n)) \Leftrightarrow ((ap c\_2Enum\_2ESUC V0m) = V1n)))))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V2p))) \Rightarrow (p (ap (ap \\
& c\_2Eprim\_rec\_2E\_3C V0m) V2p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V0m)))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2D \\
& V0n) V1m)) V0n)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1c \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0a) \\
& V1c)) V1c) = V0a))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = \\
& c\_2Enum\_2E0) \vee (V1n = c\_2Enum\_2E0))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
& c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) V0n))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2A \\
& (ap c\_2Enum\_2ESUC V2p)) V0m)) (ap (ap c\_2Earithmetic\_2E\_2A (ap \\
& c\_2Enum\_2ESUC V2p)) V1n))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V2p))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum. ((V1k = (ap ( \\
& ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A (ap (ap c\_2Earithmetic\_2EDIV \\
& V1k) V0n)) V0n)) (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n))) \wedge (p (ap \\
& (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n)) \\
& V0n))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
& \forall V2q \in ty\_2Enum\_2Enum. ((\exists V3r \in ty\_2Enum\_2Enum. ( \\
& (V1k = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A \\
& V2q) V0n)) V3r)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V3r) V0n)))) \Rightarrow ( \\
& (ap (ap c\_2Earithmetic\_2EDIV V1k) V0n) = V2q))))
\end{aligned} \tag{52}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2r \in ty\_2Enum\_2Enum. ((\exists V3q \in ty\_2Enum\_2Enum. ( \\
& (V1k = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V3q) V0n)) V2r)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2r) V0n)))) \Rightarrow ( \\
& \quad (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n) = V2r))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD \\
& \quad V1k) V0n) = V1k))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1q \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EDIV \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2A V1q) V0n)) V0n) = V1q))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V0n)) \Rightarrow (((ap (ap c\_2Earithmetic\_2EDIV V0n) V0n) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \wedge \\
& ((ap (ap c\_2Earithmetic\_2EMOD V0n) V0n) = c\_2Enum\_2E0))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1d \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& \quad V1d))) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EDIV \\
& \quad V0n) V1d)) V0n))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2y \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V0n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1x) V2y))) \Rightarrow \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2EDIV \\
& \quad V1x) V0n)) (ap (ap c\_2Earithmetic\_2EDIV V2y) V0n))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Enum\_2Enum. \\
& (\forall V2b \in ty\_2Enum\_2Enum.((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum.(((V2b = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1e0 \in ty\_2Enum\_2Enum. \\
& (\forall V2e1 \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V0n)) \Rightarrow ((p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) \\
& V1e0) V2e1)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD V1e0) V0n) = (ap (ap \\
& c\_2Earithmetic\_2EMOD V2e1) V0n))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x0 \in ty\_2Enum\_2Enum. \\
& (\forall V2x1 \in ty\_2Enum\_2Enum.(\forall V3y0 \in ty\_2Enum\_2Enum. \\
& (\forall V4y1 \in ty\_2Enum\_2Enum.((p (ap (ap (ap c\_2Earithmetic\_2EMODEQ \\
& V0n) V1x0) V2x1)) \Rightarrow ((p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) V3y0) \\
& V4y1)) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1x0) V3y0)) (ap (ap c\_2Earithmetic\_2E\_2B V2x1) V4y1))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x0 \in ty\_2Enum\_2Enum. \\
& (\forall V2x1 \in ty\_2Enum\_2Enum.(\forall V3y0 \in ty\_2Enum\_2Enum. \\
& (\forall V4y1 \in ty\_2Enum\_2Enum.((p (ap (ap (ap c\_2Earithmetic\_2EMODEQ \\
& V0n) V1x0) V2x1)) \Rightarrow ((p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) V3y0) \\
& V4y1)) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V1x0) V3y0)) (ap (ap c\_2Earithmetic\_2E\_2A V2x1) V4y1))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Enum\_2Enum.( \\
& p (ap (ap (ap c\_2Earithmetic\_2EMODEQ V0n) V1x) V1x)))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V0n)) (ap c\_2Earithmetic\_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V0n))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V1m))) \\
& (ap (ap c\_2Earithmetic\_2EMOD (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 V1m))) (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 V0n)))))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2ENUMERAL V0n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V1m))) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 V0n))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V1m))) (ap (ap c\_2Earithmetic\_2EMOD \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V1m))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V0n)))))) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V0n)) (ap c\_2Earithmetic\_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V0n))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V1m))) \\
& (ap (ap c\_2Earithmetic\_2EMOD (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT2 V1m))) (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT2 V0n)))))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2ENUMERAL V0n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V1m))) \Rightarrow (p (ap (ap (ap c\_2Earithmetic\_2EMODEQ (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 V1m))) (ap (ap c\_2Earithmetic\_2EMOD \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V1m))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V0n))))))))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (65)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \hspace{15em} (67)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \hspace{15em} (68)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (71)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (72)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge (((\neg False) \Leftrightarrow True)))) \quad (74)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (75)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (76)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (77)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (78)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))) \quad (79)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (80)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (83)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (84)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (85)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (86)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (87)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \vee ((\neg(p\ V0t1)) \wedge (\neg(p\ V1t2)))))) \quad (88)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1b \in 2. (\forall V2x \in A\_27a. (\forall V3y \in A\_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f\ V2x))\ (ap\ V0f\ V3y)))))) \quad (89)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). (\forall V2M \in \\
& \quad A\_27a. (\forall V3N \in A\_27a. (((V2M = V3N) \wedge (\forall V4x \in A\_27a. ( \\
& (V4x = V3N) \Rightarrow ((ap\ V0f\ V4x) = (ap\ V1g\ V4x)))))) \Rightarrow ((ap\ (ap\ (c\_2Ebool\_2ELET \\
& \quad A\_27a\ A\_27b)\ V0f)\ V2M) = (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27a\ A\_27b)\ V1g) \\
& \quad V3N)))))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& \quad (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& \quad (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& \quad V5y\_27)))))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in \\
& \quad A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ ( \\
& \quad ap\ V0f\ V1v))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& \quad A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& \quad V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\
& \quad (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{93}$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c\_2Ebool\_2EBOUNDED\ V0v)) \Leftrightarrow True)) \tag{94}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27c^{A\_27a}). \\
& \quad (\forall V2x \in A\_27c. ((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a) \\
& \quad V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27c}). (\forall V1g \in (A\_27c^{A\_27a}). \\
& \quad ((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\lambda V2x \in A\_27a. \\
& \quad (ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a. (ap\ V0f\ (ap\ V1g\ V3x))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27c})^{A\_27a}). (\forall V1g \in \\
& \quad (A\_27c^{A\_27a}). ((ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ A\_27c\ A\_27b)\ V0f) \\
& \quad (\lambda V2x \in A\_27a.(ap\ V1g\ V2x))) = (\lambda V3x \in A\_27a.(ap\ (ap\ V0f\ V3x) \\
& \quad (ap\ V1g\ V3x))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& \quad A\_27b. (\forall V2y \in A\_27a. ((ap\ (ap\ (ap\ (c\_2Ecombin\_2EC\ A\_27a\ A\_27b \\
& \quad A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27b^{A\_27c})^{A\_27a}). (\forall V1y \in \\
& \quad A\_27c. ((ap\ (ap\ (c\_2Ecombin\_2EC\ A\_27a\ A\_27c\ A\_27b)\ (\lambda V2x \in A\_27a. \\
& \quad (ap\ V0f\ V2x)))\ V1y) = (\lambda V3x \in A\_27a. (ap\ (ap\ V0f\ V3x)\ V1y))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI \\
& \quad A\_27a)\ V0x) = V0x))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0P \in (A\_27a^{A\_27b}). (\forall V1f \in (A\_27b^{A\_27c}). \\
& \quad (\forall V2v \in A\_27c. ((ap\ V0P\ (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27c\ A\_27b) \\
& \quad V1f)\ V2v)) = (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27c\ A\_27a)\ (ap\ (ap\ (c\_2Ecombin\_2Eo \\
& \quad A\_27c\ A\_27a\ A\_27b)\ V0P)\ V1f))\ V2v))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27a^{A\_27c})^{A\_27b}). (\forall V1v \in \\
& \quad A\_27b. (\forall V2x \in A\_27c. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27b\ ( \\
& \quad A\_27a^{A\_27c}))\ V0f)\ V1v)\ V2x) = (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27b\ A\_27a) \\
& \quad (ap\ (ap\ (c\_2Ecombin\_2EC\ A\_27b\ A\_27c\ A\_27a)\ V0f)\ V2x))\ V1v))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in \\
& \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2ELET\ A\_27a\ 2)\ V0f)\ V1v)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E\_21 \\
& \quad A\_27a)\ (ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ 2\ 2)\ (ap\ (ap\ (c\_2Ecombin\_2Eo \\
& \quad A\_27a\ (2^2)\ 2)\ c\_2Emin\_2E\_3D\_3D\_3E)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a \\
& \quad 2\ 2)\ c\_2Emarker\_2EAbbrev)\ (ap\ (ap\ (c\_2Ecombin\_2EC\ A\_27a\ A\_27a \\
& \quad 2)\ (c\_2Emin\_2E\_3D\ A\_27a))\ V1v))))\ V0f))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}), \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& \quad c\_2Elist\_2ECONS\ A\_27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& \quad A\_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a. (\forall V1a1 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V2a0\_27 \in A\_27a. (\forall V3a1\_27 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0) \\
& \quad V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = \\
& \quad V2a0\_27) \wedge (V1a1 = V3a1\_27))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Enum\_2ESUC\ c\_2Earithmetic\_2EZERO) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
& \quad c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad V1n)) = (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enum\_2ESUC\ V1n))))))
\end{aligned} \tag{106}$$



Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{109}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO)\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO) \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V0n)\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))))))))) \\
& \tag{110}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ c\_2Earithmic\_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))))))))) \\
& \tag{111}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Eprim\_rec\_2EPRE\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\
& (((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = \\
& \quad c\_2Earithmetic\_2EZERO) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
& \quad c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad V0n))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap \\
& \quad c\_2Earithmetic\_2EBIT1\ V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad V1n))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad V1n)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. ((ap\ c\_2Eprim\_rec\_2EPRE \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V2n)) = (ap\ c\_2Earithmetic\_2EBIT1\ V2n))))))
\end{aligned}
\tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmetic\_2EZERO) V0x) = c\_2Earithmetic\_2EZERO) \wedge ( \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmetic\_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (113)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V0n) \\
& V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V1m) V0n)) (ap c\_2Earithmetic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
& c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
& \hspace{15em} (114)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (115)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ c\_2Earithmetic\_2EZERO)\ V0n) = c\_2Earithmetic\_2EZERO) \wedge \\
& \quad (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ c\_2Earithmetic\_2EZERO) = \\
& \quad c\_2Earithmetic\_2EZERO) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ V1m) = (ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad (ap\ c\_2Enumeral\_2EiDUB\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m))) \\
& \quad V1m))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ V1m) = (ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m)) \\
& \quad V1m)))))))))) \\
& \hspace{15em} (116)
\end{aligned}$$

Assume the following.

$$(p\ (ap\ (c\_2Erelation\_2EWF\ ty\_2Enum\_2Enum)\ c\_2Eprim\_rec\_2E\_3C)) \quad (117)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& \quad ((p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}). \\
& \quad ((\forall V2x \in A\_27a. ((\forall V3y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow \\
& \quad (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A\_27a. (p\ (ap\ V1P\ V4x)))))) \\
& \hspace{15em} (118)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (A\_27b^{A\_27a}). (\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2y \in \\
& \quad A\_27a. (\forall V3z \in A\_27a. ((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap \\
& \quad (ap\ (c\_2Erelation\_2ERESTRICT\ A\_27a\ A\_27b)\ V0f)\ V1R)\ V3z)\ V2y) = \\
& \quad (ap\ V0f\ V2y)))))) \\
& \hspace{15em} (119)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}). (\forall V1R \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V2f \in (A\_27b^{A\_27a}). ((V2f = (ap\ (ap\ (c\_2Erelation\_2EWFREC \\
& \quad A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ V1R)) \Rightarrow \\
& \quad (\forall V3x \in A\_27a. ((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c\_2Erelation\_2ERESTRICT \\
& \quad A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))) \\
& \hspace{15em} (120)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (121)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (122)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (123)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (124)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (125)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (126)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (127)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (128)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (129)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (130)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))))) \\
& \tag{131}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{132}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{133}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{134}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{135}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{136}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty\_2Enum\_2Enum. (\forall V1r\_27 \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0r) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))))) \Rightarrow \\
& ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V1r\_27) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))))) \Rightarrow \\
& (((ap c\_2Estring\_2ECHR V0r) = (ap c\_2Estring\_2ECHR V1r\_27)) \Leftrightarrow ( \\
& V0r = V1r\_27)))))) \\
& \tag{137}
\end{aligned}$$

Assume the following.

$$(\forall V0a \in ty\_2Estring\_2Echar. ((ap c\_2Estring\_2ECHR (ap c\_2Estring\_2EORD V0a)) = V0a)) \tag{138}$$



Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty\_2Estring\_2Echar.(p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Estring\_2EORD V0c)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))))) \\
& \hspace{15em} (139)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Estring\_num\_2Es2n (c\_2Elist\_2ENIL ty\_2Estring\_2Echar)) = \\
& \quad c\_2Enum\_2E0) \wedge (\forall V0c \in ty\_2Estring\_2Echar. (\forall V1s \in \\
& \quad (ty\_2Elist\_2Elist ty\_2Estring\_2Echar). ((ap c\_2Estring\_num\_2Es2n \\
& \quad (ap (ap (c\_2Elist\_2ECONS ty\_2Estring\_2Echar) V0c) V1s)) = (ap ( \\
& \quad ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad (ap c\_2Estring\_num\_2Es2n V1s)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& \quad (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\
& \quad (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))))) \\
& \quad (ap c\_2Estring\_2EORD V0c)) (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& \quad c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\
& \hspace{15em} (140)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (ty\_2Elist\_2Elist ty\_2Estring\_2Echar). ((ap c\_2Estring\_num\_2En2s \\
& \quad (ap c\_2Estring\_num\_2Es2n V0s)) = V0s))
\end{aligned}$$