

thm\_2Esum\_2ESUM\_\_ALL\_\_CONG  
(TMH5KmQgbTzsfxQqNqCavGUHiqgiXDH6xUA)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow q Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \tag{1}$$

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{2}$$

**Definition 9** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS\_sum A\_27a A\_27b) V0e)$

**Definition 10** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum A\_27a A\_27b) V0e)$

Let  $c\_2Esum\_2ESUM\_ALL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2ESUM\_ALL\ A\_27a\ A\_27b \in (((2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)})^{(2^{A\_27b})})^{(2^{A\_27a})}) \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (16)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow ((\forall V0y \in A_{.27a}.(\forall V1x \in A_{.27a}.((ap (c_{.2Esum_{.2EINL}} A_{.27a} A_{.27b}) V0y)) \Leftrightarrow (V1x = V0y)))) \wedge (\forall V2y \in A_{.27b}.(\forall V3x \in A_{.27b}.((ap (c_{.2Esum_{.2EINR}} A_{.27a} A_{.27b}) V2y)) \Leftrightarrow (V3x = V2y)))))) \quad (17)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow ((\forall V0P \in (2^{(ty_{.2Esum_{.2Esum}} A_{.27a} A_{.27b})}).((\forall V1s \in (ty_{.2Esum_{.2Esum}} A_{.27a} A_{.27b}).(p (ap V0P V1s))) \Leftrightarrow ((\forall V2x \in A_{.27a}.(p (ap V0P (ap (c_{.2Esum_{.2EINL}} A_{.27a} A_{.27b}) V2x)))) \wedge (\forall V3y \in A_{.27b}.(p (ap V0P (ap (c_{.2Esum_{.2EINR}} A_{.27a} A_{.27b}) V3y)))))))))) \quad (18)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow ((\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\neg((ap (c_{.2Esum_{.2EINL}} A_{.27a} A_{.27b}) V0x) = (ap (c_{.2Esum_{.2EINR}} A_{.27a} A_{.27b}) V1y)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27b}).(\forall V2x \in \\
& \quad A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_ALL\ A\_27a\ A\_27b)\ V0P)\ V1Q) \\
& \quad (ap\ (c\_2Esum\_2EINL\ A\_27a\ A\_27b)\ V2x))) \Leftrightarrow (p\ (ap\ V0P\ V2x)))))) \wedge (\forall V3P \in \\
& \quad (2^{A\_27a}).(\forall V4Q \in (2^{A\_27b}).(\forall V5y \in A\_27b.((p\ (ap \\
& \quad (ap\ (ap\ (c\_2Esum\_2ESUM\_ALL\ A\_27a\ A\_27b)\ V3P)\ V4Q)\ (ap\ (c\_2Esum\_2EINR \\
& \quad A\_27a\ A\_27b)\ V5y))) \Leftrightarrow (p\ (ap\ V4Q\ V5y))))))
\end{aligned} \tag{20}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0s \in (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).(\forall V1s\_27 \in (ty\_2Esum\_2Esum \\
& \quad A\_27a\ A\_27b).(\forall V2P \in (2^{A\_27a}).(\forall V3P\_27 \in (2^{A\_27a}). \\
& \quad (\forall V4Q \in (2^{A\_27b}).(\forall V5Q\_27 \in (2^{A\_27b}).(((V0s = V1s\_27) \wedge \\
& \quad ((\forall V6a \in A\_27a.((V1s\_27 = (ap\ (c\_2Esum\_2EINL\ A\_27a\ A\_27b) \\
& \quad V6a)) \Rightarrow ((p\ (ap\ V2P\ V6a)) \Leftrightarrow (p\ (ap\ V3P\_27\ V6a)))))) \wedge (\forall V7b \in A\_27b. \\
& \quad ((V1s\_27 = (ap\ (c\_2Esum\_2EINR\ A\_27a\ A\_27b)\ V7b)) \Rightarrow ((p\ (ap\ V4Q\ V7b)) \Leftrightarrow \\
& \quad (p\ (ap\ V5Q\_27\ V7b)))))))))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_ALL\ A\_27a \\
& \quad A\_27b)\ V2P)\ V4Q)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_ALL\ A\_27a \\
& \quad A\_27b)\ V3P\_27)\ V5Q\_27)\ V1s\_27))))))
\end{aligned}$$