thm_2Esum_2ESUM__ALL__MONO (TMYeou2y8EqinPo29GhQ2KzjiyaSadrMaH5)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{1}$$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{2}$$

Definition 11 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 12 *We define* $\mathsf{c\_2Esum\_2EINL}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Esum\_2ESUM\_\_ALL : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2ESUM\_\_ALL\ A\_27a\ A\_27b \in (((2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)})^{(2^{A\_27b})})^{(2^{A\_27a})}) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0ss \in (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).((\exists V1x \in A\_27a.(V0ss = (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V1x)))\lor(\exists V2y \in A\_27b.(V0ss = (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V2y))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}((\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27b}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_\_ALL\ A\_27a\ A\_27b)\ V0P)\ V1Q)\ (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V2x))){\Leftrightarrow}(p\ (ap\ V0P\ V2x))))))\land(\forall V3P \in (2^{A\_27a}).(\forall V4Q \in (2^{A\_27b}).(\forall V5y \in A\_27b.((p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_\_ALL\ A\_27a\ A\_27b)\ V3P)\ V4Q)\ (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V5y))){\Leftrightarrow}(p\ (ap\ V4Q\ V5y)))))))) \tag{5}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0P \in (2^{A\_27a}).(\forall V1P\_27 \in (2^{A\_27a}).(\forall V2Q \in (2^{A\_27b}).(\forall V3Q\_27 \in (2^{A\_27b}).(\forall V4s \in (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).(((\forall V5x \in A\_27a.((p\ (ap\ V0P\ V5x)){\Rightarrow}(p\ (ap\ V1P\_27\ V5x))))\land(\forall V6y \in A\_27b.((p\ (ap\ V2Q\ V6y)){\Rightarrow}(p\ (ap\ V3Q\_27\ V6y))))){\Rightarrow}((p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_\_ALL\ A\_27a\ A\_27b)\ V0P)\ V2Q)\ V4s)){\Rightarrow}(p\ (ap\ (ap\ (ap\ (c\_2Esum\_2ESUM\_\_ALL\ A\_27a\ A\_27b)\ V1P\_27)\ V3Q\_27)\ V4s)))))))))$$