

thm\_2Esum\_2ESUM\_MAP\_I  
(TMWxVq66GU62LfjaofvNpWqai8obUYptGmA)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Ecombin\_2E\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 7** We define  $c\_2Ecombin\_2E\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \tag{1}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{2}$$

**Definition 11** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS\_sum$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS$

Let  $c\_2Esum\_2E\_2B\_2B : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\ & nonempty A\_27c \Rightarrow \forall A\_27d.nonempty A\_27d \Rightarrow c\_2Esum\_2E\_2B\_2B \\ A\_27a A\_27b A\_27c A\_27d \in & (((ty\_2Esum\_2Esum A\_27c A\_27d)^{(ty\_2Esum\_2Esum A\_27a A\_27b)})(A\_27d^{A\_27b})) \end{aligned} \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & (\forall V0y \in A\_27a.(\forall V1x \in A\_27a.(((ap (c\_2Esum\_2EINL \\ & A\_27a A\_27b) V1x) = (ap (c\_2Esum\_2EINL A\_27a A\_27b) V0y)) \Leftrightarrow (V1x = \\ & V0y)))) \wedge (\forall V2y \in A\_27b.(\forall V3x \in A\_27b.(((ap (c\_2Esum\_2EINR \\ & A\_27a A\_27b) V3x) = (ap (c\_2Esum\_2EINR A\_27a A\_27b) V2y)) \Leftrightarrow (V3x = \\ & V2y)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0P \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}).((\forall V1s \in \\
& \quad (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).(p\ (ap\ V0P\ V1s))) \Leftrightarrow ((\forall V2x \in \\
& \quad A\_27a.(p\ (ap\ V0P\ (ap\ (c\_2Esum\_2EINL\ A\_27a\ A\_27b)\ V2x)))) \wedge (\forall V3y \in \\
& \quad A\_27b.(p\ (ap\ V0P\ (ap\ (c\_2Esum\_2EINR\ A\_27a\ A\_27b)\ V3y))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow ((\forall V0f \in ( \\
& \quad A\_27c^{A\_27a}).(\forall V1g \in (A\_27d^{A\_27b}).(\forall V2a \in A\_27a. \\
& \quad ((ap\ (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V0f)\ V1g) \\
& \quad (ap\ (c\_2Esum\_2EINL\ A\_27a\ A\_27b)\ V2a)) = (ap\ (c\_2Esum\_2EINL\ A\_27c \\
& \quad A\_27d)\ (ap\ V0f\ V2a)))))) \wedge (\forall V3f \in (A\_27c^{A\_27a}).(\forall V4g \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5b \in A\_27b.((ap\ (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B \\
& \quad A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V3f)\ V4g)\ (ap\ (c\_2Esum\_2EINR\ A\_27a\ A\_27b) \\
& \quad V5b)) = (ap\ (c\_2Esum\_2EINR\ A\_27c\ A\_27d)\ (ap\ V4g\ V5b))))))
\end{aligned} \tag{12}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B\ A\_27a\ A\_27b\ A\_27a\ A\_27b)\ (c\_2Ecombin\_2EI \\
& \quad A\_27a))\ (c\_2Ecombin\_2EI\ A\_27b)) = (c\_2Ecombin\_2EI\ (ty\_2Esum\_2Esum \\
& \quad A\_27a\ A\_27b)))
\end{aligned}$$