

thm_2Esum_2Esum_axiom (TM- MEWs6qQJ5bZs5XRZUvkKs516ja6ryV8XP)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Definition 8 We define `c_2Ebool_2E_3F_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Ebool_2E_2F_5C } A))))$

Definition 9 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 10 We define `c_2Ebool_2E_2COND` to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (V1t1 \wedge V2t2))))$

Definition 11 We define `c_2Ecombin_2E_2Eo` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0f \in (A. 27b^{A-27c}). \lambda V1g \in (A. 27c^{A-27b}). \lambda V2h \in (A. 27a^{A-27b-27c}). \text{if } (V0f \wedge V1g) \text{ then } V2h$

Definition 12 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 13 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F_5C } V0t))))$

Definition 14 We define `c_2Esum_2EIS_SUM_REP` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (((2^{A-27b})^{A-27a})^2)$

Let $\text{ty_2Esum_2Esum} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 A1) \quad (1)$$

Let $c_2Esum_2EREP_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EREP_sum\ A_27a\ A_27b \in (((2^{A_27b})^{A_27a})^2)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \quad (2)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (3)$$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Emin_2E_40 \\
& A_27a) (\lambda V1y \in A_27a. (ap (ap (c_2Emin_2E_3D A_27a) V0x) V1y))) = \\
& V0x))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in A_27a. (\exists V1x \in \\
& A_27a. (V1x = V0a)))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0a \in (ty_2Esum_2Esum A_27a A_27b). ((ap (c_2Esum_2EABS_sum \\
& A_27a A_27b) (ap (c_2Esum_2EREP_sum A_27a A_27b) V0a)) = V0a)) \wedge \\
& (\forall V1r \in (((2^{A_27b})^{A_27a})^2). ((p (ap (c_2Esum_2EIS_SUM_REP \\
& A_27a A_27b) V1r)) \Leftrightarrow ((ap (c_2Esum_2EREP_sum A_27a A_27b) (ap (\\
& c_2Esum_2EABS_sum A_27a A_27b) V1r)) = V1r))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap\ (c_2Esum_2EINL\ A_27a \\ & A_27b)\ V0x) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\ & \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27b. (\forall V1y \in A_27b. (((ap\ (c_2Esum_2EINR\ A_27a \\ & A_27b)\ V0x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\ & \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0v1 \in A_27a. (\forall V1v2 \in A_27b. (\neg((ap\ (c_2Esum_2EINR \\ & A_27a\ A_27b)\ V1v2) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V0v1)))))) \\ & \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27a}). (\forall V1g \in (A_27c^{A_27b}). \\ & (p\ (ap\ (c_2Ebool_2E_3F_21\ (A_27c^{(ty_2Esum_2Esum\ A_27a\ A_27b)})) \\ & (\lambda V2h \in (A_27c^{(ty_2Esum_2Esum\ A_27a\ A_27b)}). (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ (c_2Emin_2E_3D\ (A_27c^{A_27a}))\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a \\ & A_27c\ (ty_2Esum_2Esum\ A_27a\ A_27b))\ V2h)\ (c_2Esum_2EINL\ A_27a \\ & A_27b))))\ V0f))\ (ap\ (ap\ (c_2Emin_2E_3D\ (A_27c^{A_27b}))\ (ap\ (ap\ (c_2Ecombin_2Eo \\ & A_27b\ A_27c\ (ty_2Esum_2Esum\ A_27a\ A_27b))\ V2h)\ (c_2Esum_2EINR \\ & A_27a\ A_27b))))\ V1g)))))) \\ & \end{aligned}$$