

thm_2Esum_2Esum__case__cong
(TMHcEUqLt6BrqBiAGFPDAJ6gca35D6dUbx3)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 3 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a V0P))$

Definition 5 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \tag{1}$$

Definition 8 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{2}$$

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Let $c_2Esum_2Esum_CASE : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow c_2Esum_2Esum_CASE A_27a A_27b A_27c \in (((A_27c^{(A_27c^{A_27b})})^{(A_27c^{A_27a})})^{(ty_2Esum_2Esum_CASE A_27a A_27b A_27c)})) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0ss \in (ty_2Esum_2Esum A_27a A_27b).((\exists V1x \in A_27a. \\ & (V0ss = (ap (c_2Esum_2EINL A_27a A_27b) V1x))) \vee (\exists V2y \in A_27b. \\ & (V0ss = (ap (c_2Esum_2EINR A_27a A_27b) V2y)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow ((\forall V0x \in A_27a.(\forall V1f \in (A_27c^{A_27a}). \\ & (\forall V2f1 \in (A_27c^{A_27b}).((ap (ap (ap (c_2Esum_2Esum_CASE \\ & A_27a A_27b A_27c) (ap (c_2Esum_2EINL A_27a A_27b) V0x)) V1f) V2f1) = \\ & (ap V1f V0x)))))) \wedge (\forall V3y \in A_27b.(\forall V4f \in (A_27c^{A_27a}). \\ & (\forall V5f1 \in (A_27c^{A_27b}).((ap (ap (ap (c_2Esum_2Esum_CASE \\ & A_27a A_27b A_27c) (ap (c_2Esum_2EINR A_27a A_27b) V3y)) V4f) V5f1) = \\ & (ap V5f1 V3y))))))))) \end{aligned} \quad (5)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow (\forall V0f_27 \in (A_27c^{A_27a}).(\forall V1f1_27 \in \\ & (A_27c^{A_27b}).(\forall V2M \in (ty_2Esum_2Esum A_27a A_27b).(\forall V3M_27 \in \\ & (ty_2Esum_2Esum A_27a A_27b).(\forall V4f \in (A_27c^{A_27a}).(\forall V5f1 \in \\ & (A_27c^{A_27b}).(((V2M = V3M_27) \wedge ((\forall V6x \in A_27a.((V3M_27 = \\ & (ap (c_2Esum_2EINL A_27a A_27b) V6x)) \Rightarrow ((ap V4f V6x) = (ap V0f_27 \\ & V6x)))))) \wedge (\forall V7y \in A_27b.((V3M_27 = (ap (c_2Esum_2EINR A_27a \\ & A_27b) V7y)) \Rightarrow ((ap V5f1 V7y) = (ap V1f1_27 V7y))))))) \Rightarrow ((ap (ap (ap \\ & (c_2Esum_2Esum_CASE A_27a A_27b A_27c) V2M) V4f) V5f1) = (ap (ap \\ & (ap (c_2Esum_2Esum_CASE A_27a A_27b A_27c) V3M_27) V0f_27) V1f1_27))))))))) \end{aligned}$$