

thm_2Esum__num_2ESUM__LESS (TMQMm- BaTr6SFFd39MxX8YnSL4xMhTNUEt9z)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21) 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $c_2Esum_num_2EGSUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2EGSUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (9)$$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & \forall V2n \in ty_2Enum_2Enum. (\forall V3f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). \\ & ((\exists V4q \in ty_2Enum_2Enum. ((p (ap (ap\ c.2Earithmetic_2E_3C_3D \\ & (ap (ap\ c.2Earithmetic_2E_2B\ V1m) V0p)) V4q)) \wedge ((p (ap (ap\ c.2Eprim_rec_2E_3C \\ & V4q) (ap (ap\ c.2Earithmetic_2E_2B\ V2n) V0p))) \wedge (\neg((ap\ V3f\ V4q) = \\ & c.2Enum_2E0)))))) \Leftrightarrow (p (ap (ap\ c.2Eprim_rec_2E_3C (ap (ap\ c.2Esum_num_2EGSUM \\ & (ap (ap (c.2Epair_2E_2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum) V0p) V1m)) \\ & V3f)) (ap (ap\ c.2Esum_num_2EGSUM (ap (ap (c.2Epair_2E_2C\ ty_2Enum_2Enum \\ & ty_2Enum_2Enum) V0p) V2n)) V3f)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). \\ & ((ap (ap\ c.2Esum_num_2ESUM\ V0m) V1f) = (ap (ap\ c.2Esum_num_2EGSUM \\ & (ap (ap (c.2Epair_2E_2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum) c.2Enum_2E0) \\ & V0m)) V1f)))) \end{aligned} \quad (14)$$

Theorem 1

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). ((\exists V3q \in \\ & ty_2Enum_2Enum. ((p (ap (ap\ c.2Earithmetic_2E_3C_3D\ V0m) V3q)) \wedge \\ & ((p (ap (ap\ c.2Eprim_rec_2E_3C\ V3q) V1n)) \wedge (\neg((ap\ V2f\ V3q) = c.2Enum_2E0)))))) \Leftrightarrow \\ & (p (ap (ap\ c.2Eprim_rec_2E_3C (ap (ap\ c.2Esum_num_2ESUM\ V0m) \\ & V2f)) (ap (ap\ c.2Esum_num_2ESUM\ V1n) V2f)))))) \end{aligned}$$