thm_2Etc_2ERTC__INSERT (TMZVsbQ1fYwXLzGmPQocNBp5TCL38f4KsAT)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 11 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 12 *We define* c_2Erelation_2Etransitive *to be* $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ ($c_2Ebool_2

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 15 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

Definition 16 *We define* c_2Etc_2E_5E_7C *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 17 *We define* c_2Etc_2E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 18 *We define* c_2Etc_2E_5E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).(a$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{10}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{14}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\wedge(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\wedge((p\ V0A)\vee(p\ V2C))))))) \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s)))\Leftrightarrow((V0x = V1y)\vee(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s))\Rightarrow((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s) = V1s)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R) V1x)\ V1x)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V1x)\ V2y)))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1P \in ((2^{A\_27a})^{A\_27a}).(((\forall V2x \in A\_27a.(p\ (ap\ (ap\ V1P\ V2x)\ V2x))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in A\_27a.(((p\ (ap\ (ap\ V0R\ V3x)\ V4y)) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V4y)\ V5z)) \wedge (p\ (ap\ (ap\ V1P\ V4y)\ V5z)))) \Rightarrow (p\ (ap\ (ap\ V1P\ V3x)\ V5z))))))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V6x)\ V7y)) \Rightarrow (p\ (ap\ (ap\ V1P\ V6x)\ V7y)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1P \in ((2^{A\_27a})^{A\_27a}).(((\forall V2x \in A\_27a.(p\ (ap\ (ap\ V1P\ V2x)\ V2x))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in A\_27a.(((p\ (ap\ (ap\ V1P\ V3x)\ V4y)) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V3x)\ V4y)) \wedge (p\ (ap\ (ap\ V0R\ V4y)\ V5z)))) \Rightarrow (p\ (ap\ (ap\ V1P\ V3x)\ V5z))))))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V6x)\ V7y)) \Rightarrow (p\ (ap\ (ap\ V1P\ V6x)\ V7y)))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V1x)\ V2y)) \Leftrightarrow ((V1x = V2y) \vee (\exists V3u \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V3u)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V3u)\ V2y))))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V1x)\ V2y)) \Leftrightarrow ((V1x = V2y) \vee (\exists V3u \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V1x)\ V3u)) \wedge (p\ (ap\ (ap\ V0R\ V3u)\ V2y))))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).\ (\forall V1Q \in ((2^{A\_27a})^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27a.((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((p\ (ap\ (ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V1Q\ V4x)\ V5y)))))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V0R)\ V2x)\ V3y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V1Q)\ V2x)\ V3y))))))))) \tag{26}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).\ (\forall V1s \in (2^{A\_27a}).(\forall V2a \in A\_27a.(\forall V3w \in A\_27a.\ (\forall V4z \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2a)\ V1s)))\ V3w)\ V4z)) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ V1s))\ V3w)\ V4z)) \lor (((V2a = V3w) \lor (\exists V5x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ V1s)) \land ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ V1s))\ V3w)\ V5x)) \land (p\ (ap\ (ap\ V0R\ V5x)\ V2a)))))) \land ((V2a = V4z) \lor (\exists V6y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6y)\ V1s)) \land ((p\ (ap\ (ap\ V0R\ V2a)\ V6y)) \land (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ V1s))\ V6y)\ V4z))))))))))))))$$