thm_2Etc_2EsubTC__EMPTY (TMdHgirYTbvp1f6wCk2yfMNzvQEJyLddjGu)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 7 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 8 *We define* c_2Erelation_2EEMPTY__REL *to be* $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27a.$c_2Eb

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 10 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 11 *We define* c_2Etc_2E_5E_7C *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 12 *We define* c_2Etc_2E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 13 *We define* c_2Etc_2E_5E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).(a$

Definition 14 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

Definition 15 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 17 *We define* c_2Erelation_2EO *to be* $\lambda A\_27g : \iota.\lambda A\_27h : \iota.\lambda A\_27k : \iota.\lambda V0R1 \in ((2^{A\_27k})^{A\_27h})$

Definition 18 *We define* c_2Erelation_2ERUNION *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27b})^{A\_27a}).\lambda V1$

Definition 19 *We define* c_2Etc_2EsubTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2x$

Assume the following.

$$True \tag{1}$$

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$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0R \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C}\ A\_27a)\ V0R)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0R \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ V0R)\ (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a)) = (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a)))\wedge(\forall V1R \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a))\ V1R) = (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a)))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Etc\_2EsubTC}\ A\_27a)\ V0R)\ V1s) = (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERUNION}\ A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ V1s)))\ V1s))\ V0R)))))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).\ ((ap\ (ap\ (\mathsf{c\_2Etc\_2EsubTC}\ A\_27a)\ V0R)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = V0R))$$