thm_2Etc_2EsubTC__thm (TMLEVuMN-SiYdS3iUN2uQ3gWwdTkAqot6pQg)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Erelation_2EO *to be* $\lambda A\_27g : \iota.\lambda A\_27h : \iota.\lambda A\_27k : \iota.\lambda V0R1 \in ((2^{A\_27k})^{A\_27h})$

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Erelation_2ERUNION *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27b})^{A\_27a}).\lambda V1$

Definition 13 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 14 *We define* c_2Etc_2E_5E_7C *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 15 *We define* c_2Etc_2E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2a$

Definition 16 *We define* c_2Etc_2E_5E_7C_5E *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).(a$

Definition 17 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

Definition 18 *We define* c_2Etc_2EsubTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).\lambda V2x$

Assume the following.

$$True \quad (1)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (2)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (4)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}).((\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ (ap\ V1R\ V2x)\ V3y))) \Rightarrow (p\ (ap\ V0P\ V3y))))) \Rightarrow (\forall V4x \in A\_27a.(\forall V5y \in A\_27a.(((p\ (ap\ V0P\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ V1R)\ V4x)\ V5y))) \Rightarrow (p\ (ap\ V0P\ V5y)))))))) \quad (6)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Etc\_2EsubTC}\ A\_27a)\ V0R)\ V1s) = (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERUNION}\ A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ V0R)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ERTC}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Etc\_2E\_5E\_7C\_5E}\ A\_27a)\ V0R)\ V1s)))\ V1s))\ V0R))))))$$