thm_2EternaryComparisons_2Edatatype__ordering (TMdV5D98ZX3UbMpmvF3K5SuRhZo6iKUv1NY)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (1)$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \quad (2)$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (3)$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2EDATATYPE *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((p\ (ap\ (\mathsf{c\_2Ebool\_2EDATATYPE}\ A\_27a)\ V0x)) \Leftrightarrow True)) \quad (6)$$

Theorem 1

$(\forall V0ordering \in (((2^{ty\_2EternaryComparisons\_2Eordering})^{ty\_2EternaryComparisons\_2Eordering})^{ty\_2EternaryComp}$
$(p\ (ap\ (\mathsf{c\_2Ebool\_2EDATATYPE}\ 2)\ (ap\ (ap\ (ap\ V0ordering\ c\_2EternaryComparisons\_2ELI$
$c\_2EternaryComparisons\_2EEQUAL)\ c\_2EternaryComparisons\_2EGREATER))))$