# thm_2EternaryComparisons_2Enum2ordering__ONTO (TM-Gai2knkuJMTMS5EnuNnMQFJBeg4ygkBiq)

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**Definition 1** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

**Definition 4** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 5** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27a}$

**Definition 6** *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t)).$

**Definition 7** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 8** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2EF

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

**Definition 9** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

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**Definition 10** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

**Definition 11** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(6)$$

**Definition 12** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 13** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

**Definition 14** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

**Definition 15** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \qquad (7)$$

Let $c\_2EternaryComparisons\_2Eordering2num : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Eordering2num \in (ty\_2Enum\_2Enum^{ty\_2EternaryComparisons\_2Eordering})$$
$$(8)$$

Let $c\_2EternaryComparisons\_2Enum2ordering : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Enum2ordering \in (ty\_2EternaryComparisons\_2Eordering^{ty\_2Enum\_2Enum}$$
$$(9)$$

Assume the following.

$$True \qquad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \qquad (11)$$
$$p\ V0t)))))))$$

Assume the following.

$$((\forall V0a \in ty\_2EternaryComparisons\_2Eordering.((ap\ c\_2EternaryComparisons\_2Enum2ordering$$
$$(ap\ c\_2EternaryComparisons\_2Eordering2num\ V0a)) = V0a)) \wedge (\forall V1r \in$$
$$ty\_2Enum\_2Enum.((p\ (ap\ (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V2n)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))) \ V1r)) \Leftrightarrow$$
$$((ap\ c\_2EternaryComparisons\_2Eordering2num\ (ap\ c\_2EternaryComparisons\_2Enum2ordering$$
$$V1r)) = V1r))))$$

$$(12)$$

**Theorem 1**

$$(\forall V0a \in ty\_2EternaryComparisons\_2Eordering.(\exists V1r \in$$
$$ty\_2Enum\_2Enum.((V0a = (ap\ c\_2EternaryComparisons\_2Enum2ordering$$
$$V1r)) \wedge (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1r)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))))))$$