

thm_2EternaryComparisons_2Eordering2num__ONTO (TMXm7qfSTTxN6DPnRp6sn98bmqyudehe7y2)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B (ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (7)$$

Let $c_2EternaryComparisons_2Eordering2num : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2Eordering2num \in (ty_2Enum_2Enum^{ty_2EternaryComparisons_2Eordering})^{ty_2EternaryComparisons_2Eordering} \quad (8)$$

Let $c_2EternaryComparisons_2Enum2ordering : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2Enum2ordering \in (ty_2EternaryComparisons_2Eordering^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0a \in ty_2EternaryComparisons_2Eordering.((ap\ c_2EternaryComparisons_2Enum2ordering \\ & \quad (ap\ c_2EternaryComparisons_2Eordering2num\ V0a) = V0a)) \wedge (\forall V1r \in \\ & \quad ty_2Enum_2Enum.((p\ (ap\ (\lambda V2n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Eprim_rec_2E_3C \\ & \quad \quad V2n) (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT1 \\ & \quad \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V1r)) \Leftrightarrow \\ & \quad ((ap\ c_2EternaryComparisons_2Eordering2num (ap\ c_2EternaryComparisons_2Enum2ordering \\ & \quad \quad V1r)) = V1r)))) \end{aligned} \quad (11)$$

Theorem 1

$(\forall V0r \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C$
 $V0r) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1$
 $(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Leftrightarrow (\exists V1a \in$
 $ty_2EternaryComparisons_2Eordering.(V0r = (ap c_2EternaryComparisons_2Eordering2num$
 $V1a))))))$