

# thm\_2Etopology\_2ECLOSED\_\_IN\_\_DIFF (TMd- kWaMkZc2QZUx4Jq9w4mfo5NYQwE98XMt)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epair\_2EABS\_prod A.27a A.27b \in ((ty\_2Epair\_2Eprod A.27a A.27b)^{(2^{A-27b} A-27a)}) \quad (2)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2Ebool\_2E\_21 2) (c\_2Epair\_2EABS\_prod V0x V1y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Epred\_set\_2EGSPEC A.27a A.27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A.27a 2)^{A-27b}}) \quad (3)$$

**Definition 11** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Epred\_set\_2EINTER) V0s V1t)$

**Definition 12** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Epred\_set\_2EDIFF) V0s V1t)$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (4)$$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Etopology\_2Eopen\_in A.27a \in ((2^{(2^{A-27a})})^{(ty\_2Etopology\_2Etopology A.27a)}) \quad (5)$$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40) V0P)))$

**Definition 15** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A.27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c\_2Epred\_set\_2EBIGUNION) V0P)$

**Definition 16** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A.27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology A.27a)$

**Definition 17** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Epred\_set\_2ESUBSET) V0s V1t)$

**Definition 18** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A.27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology A.27a)$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (11)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t)))))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap (ap (c\_2Ebool\_2EIN A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t)))))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap (ap (c\_2Ebool\_2EIN A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (\neg(p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A\_27a). (\forall V1s \in (2^{A\_27a}). ((p (ap (ap (c\_2Etopology\_2Eclosed\_in \\ A\_27a) V0top) V1s)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET \ A\_27a) \\ V1s) (ap (c\_2Etopology\_2Etopospace \ A\_27a) V0top)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A\_27a). (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). (((p \\ (ap (ap (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) V1s)) \wedge (p (ap \\ (ap (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) V2t))) \Rightarrow (p (ap (ap \\ (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) (ap (ap (c\_2Epred\_set\_2EINTER \\ A\_27a) V1s) V2t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A\_27a). (\forall V1s \in (2^{A\_27a}). ((p (ap (ap (c\_2Etopology\_2Eopen\_in \\ A\_27a) V0top) V1s)) \Leftrightarrow ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \ A\_27a) \\ V1s) (ap (c\_2Etopology\_2Etopospace \ A\_27a) V0top))) \wedge (p (ap (ap \\ (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) (ap (ap (c\_2Epred\_set\_2EDIFF \\ A\_27a) (ap (c\_2Etopology\_2Etopospace \ A\_27a) V0top)) V1s)))))) \end{aligned} \quad (35)$$

### Theorem 1

$$\begin{aligned} \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A\_27a). (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). (((p \\ (ap (ap (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) V1s)) \wedge (p (ap \\ (ap (c\_2Etopology\_2Eopen\_in \ A\_27a) V0top) V2t))) \Rightarrow (p (ap (ap \\ (c\_2Etopology\_2Eclosed\_in \ A\_27a) V0top) (ap (ap (c\_2Epred\_set\_2EDIFF \\ A\_27a) V1s) V2t)))))) \end{aligned}$$