

thm_2Etopology_2EHULL_IMAGE_GALOIS (TMLaSf7uFjsvzG9dMSinVKfmykgbhtKNhVN)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in$

Definition 12 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2Epred_set_2EIMAGE$

Definition 13 We define $c_2Etopology_2Ehull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).\lambda V1s \in (2^{A-27a}).(ap (c_2Epred_set_2EBIGINTER$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (6) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ & (2^{A-27a}).(((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V0s) V1t)) \wedge \\ & (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V1t) V0s))) \Rightarrow (V0s = V1t)))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(\forall V1s \in \\ (2^{A-27a}).(p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V1s)\ (ap\ (\\ ap\ (c.2Etopology_2Ehull\ A.27a)\ V0P)\ V1s)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(\forall V1s \in \\ (2^{A-27a}).(\forall V2t \in (2^{A-27a}).(((p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET \\ A.27a)\ V1s)\ V2t)) \wedge (p\ (ap\ V0P\ V2t)))) \Rightarrow (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET \\ A.27a)\ (ap\ (ap\ (c.2Etopology_2Ehull\ A.27a)\ V0P)\ V1s))\ V2t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(\forall V1f \in \\ (A.27a^{A-27a}).(\forall V2s \in (2^{A-27a}).(((p\ (ap\ V0P\ (ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ V2s))) \wedge (\forall V3s \in (2^{A-27a}).((p\ (ap\ V0P\ V3s)) \Rightarrow (\\ p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ V3s)))))) \Rightarrow \\ (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ V2s))) \\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ (ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ V2s)))))) \end{aligned} \quad (15)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(\forall V1f \in \\ (A.27a^{A-27a}).(\forall V2g \in (A.27a^{A-27a}).(\forall V3s \in (2^{A-27a}). \\ (((\forall V4s \in (2^{A-27a}).(p\ (ap\ V0P\ (ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ V4s)))) \wedge ((\forall V5s \in (2^{A-27a}).((p\ (ap\ V0P\ V5s)) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ V5s)))))) \wedge \\ ((\forall V6s \in (2^{A-27a}).((p\ (ap\ V0P\ V6s)) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred_set.2EIMAGE \\ A.27a\ A.27a)\ V2g)\ V6s)))))) \wedge (\forall V7s \in (2^{A-27a}).(\forall V8t \in \\ (2^{A-27a}).((p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V7s)\ (ap \\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V2g)\ V8t))) \Leftrightarrow (p\ (ap\ (ap \\ (c.2Epred_set.2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE \\ A.27a\ A.27a)\ V1f)\ V7s))\ V8t)))))) \Rightarrow ((ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ V3s)) = \\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27a)\ V1f)\ (ap\ (ap\ (c.2Etopology_2Ehull \\ A.27a)\ V0P)\ V3s)))))) \end{aligned}$$