

thm_2Etopology_2EHULL__INDUCT (TMa81yBXag1jD5Co5wGQ9jC9iQQjtFG41mM)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 12 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EGSPEC\ V0s\ V1t))$

Definition 13 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ V0P))$

Definition 14 We define $c_2Etopology_2Ehull$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epred_set_2EBIGINTER\ V0P\ V1s))$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. (((\forall V1x \in \\ A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t)))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (11)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27}) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (12)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (13)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2E}pair_{.2E}prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V1v) (ap (c_{.2E}pred_{.set}_{.2EG}SPEC A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} 2) V1v) c_{.2E}bool_{.2ET}) = (ap V0f V2x)))))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}).(\forall V1s \in (2^{A_{.27a}}).(\forall V2t \in (2^{A_{.27a}}).(((p (ap (ap (c_{.2E}pred_{.set}_{.2ES}SUBSET A_{.27a}) V1s) V2t)) \wedge (p (ap V0P V2t))) \Rightarrow (p (ap (ap (c_{.2E}pred_{.set}_{.2ES}SUBSET A_{.27a}) (ap (ap (c_{.2E}topology_{.2E}hull A_{.27a}) V0P) V1s)) V2t)))))) \quad (16)$$

Theorem 1

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}).(\forall V1p \in (2^{A_{.27a}}).(\forall V2s \in (2^{A_{.27a}}).(((\forall V3x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V3x) V2s)) \Rightarrow (p (ap V1p V3x)))) \wedge (p (ap V0P (ap (c_{.2E}pred_{.set}_{.2EG}SPEC A_{.27a} A_{.27a}) (\lambda V4x \in A_{.27a}.(ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} 2) V4x) (ap V1p V4x)))))) \Rightarrow (\forall V5x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V5x) (ap (ap (c_{.2E}topology_{.2E}hull A_{.27a}) V0P) V2s))) \Rightarrow (p (ap V1p V5x)))))))))) \quad (17)$$