

# thm\_2Etopology\_2EHULL\_\_SUBSET (TMHc8wNt5GZynBgPnMmFriVvpEzqQkPZw2k)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}}))$

**Definition 4** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

**Definition 9** We define `c_2Ebool_2EIN` to be  $\lambda A_{27a} : \iota.(\lambda V0x \in A_{27a}.(\lambda V1f \in (2^{A_{27a}}).(ap V1f V0x)))$

**Definition 10** We define `c_2Epred__set_2ESUBSET` to be  $\lambda A_{27a} : \iota.\lambda V0s \in (2^{A_{27a}}).\lambda V1t \in (2^{A_{27a}}).(ap ($

**Definition 11** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let `c_2Epair_2EABS__prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2EABS__prod A_{27a} A_{27b} \in ((ty_2Epair_2Eprod A_{27a} A_{27b})^{(2^{A_{27b}})^{A_{27a}}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 13** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})})$ .

**Definition 14** We define  $c\_2Etopology\_2Ehull$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})})$ .

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p V0t) \Leftrightarrow (p V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( \\ p V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\ (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in A.27b. (((ap (ap (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap (ap (c.2Epair.2E.2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0f \in ((ty.2Epair.2Eprod\ A.27a\ 2)^{A-27b}). (\forall V1v \in A.27a. ((p (ap (ap (c.2Ebool.2EIN\ A.27a)\ V1v) (ap (c.2Epred.2E.set.2EGSPEC\ A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b. ((ap (ap (c.2Epair.2E.2C\ A.27a\ 2)\ V1v)\ c.2Ebool.2ET) = (ap\ V0f\ V2x)))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1B \in (2^{(2^{A-27a})}). ((p (ap (ap (c.2Ebool.2EIN\ A.27a)\ V0x) (ap (c.2Epred.2E.set.2EBIGINTER\ A.27a)\ V1B))) \Leftrightarrow (\forall V2P \in (2^{A-27a}). ((p (ap (ap (c.2Ebool.2EIN\ (2^{A-27a})\ V2P)\ V1B)) \Rightarrow (p (ap (ap (c.2Ebool.2EIN\ A.27a)\ V0x)\ V2P)))))) \quad (15)$$

### Theorem 1

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}). (\forall V1s \in (2^{A-27a}). (p (ap (ap (c.2Epred.2E.set.2ESUBSET\ A.27a)\ V1s) (ap (ap (c.2Etopology.2Ehull\ A.27a)\ V0P)\ V1s))))))$$