

thm_2Etopology_2EHULL__UNION__SUBSET
(TMVqf68xZXLgiWXwf1ZyV868qjmzxQRf38T)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2EIN$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 11 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Epred_set_2EUNION A_27a) V0s V1t)$

Definition 12 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Epred_set_2ESUBSET A_27a) V0s V1t)$

Definition 13 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2Epred_set_2EBIGINTER A_27a) V0P)$

Definition 14 We define $c_2Etopology_2Ehull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).\lambda V1s \in (2^{A-27a}).(ap (c_2Etopology_2Ehull A_27a) V0P V1s)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V1x))) \Leftrightarrow (p V0t))) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (6) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ & (2^{A-27a}).(p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V0s) (ap (ap (c_2Epred_set_2EUNION A_27a) V0s) V1t)))))) \wedge (\forall V2s \in \\ & (2^{A-27a}).(\forall V3t \in (2^{A-27a}).(p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V2s) (ap (ap (c_2Epred_set_2EUNION A_27a) V3t) V2s)))))) \quad (8) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ & (2^{A-27a}).(\forall V2u \in (2^{A-27a}).((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) (ap (ap (c_2Epred_set_2EUNION A_27a) V0s) V1t)) V2u)) \Leftrightarrow \\ & ((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V0s) V2u)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V1t) V2u)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(\forall V1s \in \\ & (2^{A-27a}).(\forall V2t \in (2^{A-27a}).((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V1s) V2t)) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) (ap (c_2Etopology_2Ehull A_27a) V0P) V1s)) (ap (ap (c_2Etopology_2Ehull A_27a) V0P) V2t)))))) \quad (10) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A.27a})}). (\forall V1s \in \\ & (2^{A.27a}). (\forall V2t \in (2^{A.27a}). (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET \\ & A.27a)\ (ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ (ap\ (ap\ (c.2Etopology.2Ehull \\ & A.27a)\ V0P)\ V1s))\ (ap\ (ap\ (c.2Etopology.2Ehull\ A.27a)\ V0P)\ V2t))) \\ & (ap\ (ap\ (c.2Etopology.2Ehull\ A.27a)\ V0P)\ (ap\ (ap\ (c.2Epred_set.2EUNION \\ & A.27a)\ V1s)\ V2t)))))) \end{aligned}$$