

thm_2Etopology_2EOPEN__IN__BIGINTER (TMXJ7BhwVofsQ9JkYMzsu1S8vZ1WXAoQKqn)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_7E (c_2Epair_2EABS_prod A_27a A_27b) V0x) V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 11 We define $c_2\text{Epred_set_2EINSERT}$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E21 (c_2Ebool_2E21)))$

Definition 12 We define $c_2\text{Epred_set_2EEMPTY}$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Definition 13 We define $c_2\text{Epred_set_2EFINITE}$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E21 (2^{A_27a})))$

Definition 14 We define $c_2\text{Epred_set_2EUNIV}$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2E)$.

Definition 15 We define $c_2\text{Epred_set_2EBIGINTER}$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set_2EEMPTY))$

Definition 16 We define $c_2\text{Epred_set_2EINTER}$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E21 (c_2Ebool_2E21)))$

Let $ty_2\text{Etopology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2\text{Etopology_2Etopology} A0) \quad (4)$$

Let $c_2\text{Etopology_2Eopen_in} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2\text{Etopology_2Eopen_in} A_27a \in ((2^{(2^{A_27a})}) (ty_2\text{Etopology_2Etopology} A_27a)) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg (p V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V1x)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (12)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow ((\forall V0s \in (2^{A.27a}).((ap (ap (c.2Epred_set.2EINSERT A.27a) (c.2Epred_set.2EUNIV A.27a)) V0s) = V0s)) \wedge (\forall V1s \in (2^{A.27a}).((ap (ap (c.2Epred_set.2EINSERT A.27a) V1s) (c.2Epred_set.2EUNIV A.27a)) = V1s))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in (2^{A.27a}).(\neg((ap (ap (c.2Epred_set.2EINSERT A.27a) V0x) V1s) = (c.2Epred_set.2EEMPTY A.27a)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ A_27a. (\forall V2s \in (2^{A_27a}). ((\forall V3x \in A_27a. ((p\ (ap\ (ap \\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a) \\ V1a)\ V2s))) \Rightarrow (p\ (ap\ V0P\ V3x)))) \Leftrightarrow ((p\ (ap\ V0P\ V1a)) \wedge (\forall V4x \in A_27a. \\ ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ V4x)))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((\\ (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\ ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A_27a. ((\neg (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1B \in \\ (2^{(2^{A_27a})}). ((ap\ (c_2Epred_set_2EBIGINTER\ A_27a)\ (ap\ (ap \\ (c_2Epred_set_2EINSERT\ (2^{A_27a})\ V0P)\ V1B)) = (ap\ (ap\ (c_2Epred_set_2EINTER \\ A_27a)\ V0P)\ (ap\ (c_2Epred_set_2EBIGINTER\ A_27a)\ V1B)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Epred_set_2EBIGINTER \\ A_27a)\ (c_2Epred_set_2EEMPTY\ (2^{A_27a}))) = (c_2Epred_set_2EUNIV \\ A_27a)) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p \\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ V0top)\ V1s)) \wedge (p\ (ap\ (ap \\ (c_2Etopology_2Eopen_in\ A_27a)\ V0top)\ V2t))) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in \\ A_27a)\ V0top)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V1s)\ V2t)))))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a). (\forall V1s \in (2^{(2^{A_27a})}). ((p\ (ap\ (c_2Epred_set_2EFINITE \\ (2^{A_27a})\ V1s)) \wedge ((\neg (V1s = (c_2Epred_set_2EEMPTY\ (2^{A_27a})))) \wedge \\ (\forall V2t \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a}) \\ V2t)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ V0top)\ V2t)))))) \Rightarrow \\ (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ V0top)\ (ap\ (c_2Epred_set_2EBIGINTER \\ A_27a)\ V1s)))))) \end{aligned}$$