

thm_2Etopology_2EOPEN__OWN__NEIGH
(TMLcXWLH-
LEzgCCX5iSFZZKqgiAANXe8P1Ak)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF } V0t))$

Definition 7 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Definition 8 We define `c_2Epred_set_2ESUBSET` to be $\lambda A. 27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Let `ty_2Etopology_2Etopology` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Etopology_2Etopology } A0) \quad (1)$$

Let `c_2Etopology_2Eopen_in` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Etopology_2Eopen_in } A. 27a \in ((2^{(2^{A-27a})}) (\text{ty_2Etopology_2Etopology } A. 27a)) \quad (2)$$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } V0P))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}})^{A_27a}) \quad (4)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Etopology_2Eneigh : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eneigh\ A_27a \in ((2^{(ty_2Epair_2Eprod\ (2^{A_27a})\ A_27a)}\ (ty_2Etopology_2Etopology\ A_27a))) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V0s))) \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology\ A_27a).(\forall V1N \in (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap \\ & (c_2Etopology_2Eneigh\ A_27a)\ V0top)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ A_27a)\ V1N)\ V2x))) \Leftrightarrow (\exists V3P \in (2^{A_27a}).((p\ (ap\ (ap \\ & (c_2Etopology_2Eopen_in\ A_27a)\ V0top)\ V3P)) \wedge ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V3P)\ V1N)) \wedge (p\ (ap\ V3P\ V2x)))))))))) \end{aligned} \quad (10)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0S_27 \in (2^{A_27a}).(\forall V1top \in \\ & (ty_2Etopology_2Etopology\ A_27a).(\forall V2x \in A_27a.(((p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ V1top)\ V0S_27)) \wedge (p\ (ap \\ & V0S_27\ V2x))) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eneigh\ A_27a)\ V1top)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ A_27a)\ V0S_27)\ V2x)))))) \end{aligned}$$