

thm_2Etopology_2EOPEN__UNOPEN
 (TMbUDgMd-
 SjcvrT58VGJDgtwM4NWTWMwMCAAt)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 10 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ ($

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if}\ (\exists x \in A. p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x. x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2E$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (4)$$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (7)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))) \quad (10)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap\ (ap\ (c_2Epair_2E_2C\ A_{.27a}\ A_{.27b})\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_{.27a}\ A_{.27b})\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0f \in ((ty_2Epair_2Eprod\ A_{.27a}\ 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC\ A_{.27a}\ A_{.27b})\ V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap\ (ap\ (c_2Epair_2E_2C\ A_{.27a}\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \quad (16)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_{.27a})\ V0s)\ V1t)) \Leftrightarrow (\forall V2x \in A_{.27a}.((p\ (ap\ V0s\ V2x)) \Rightarrow (p\ (ap\ V1t\ V2x)))))) \quad (17)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_{.27a})\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_{.27a})\ V1t)\ V0s))) \Leftrightarrow (V0s = V1t)))) \quad (18)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1sos \in (2^{(2^{A_{.27a}})}).((p\ (ap\ (ap\ (c_2Epred_set_2EBIGUNION\ A_{.27a})\ V1sos)\ V0x)) \Leftrightarrow (\exists V2s \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_{.27a}})\ V2s)\ V1sos)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\
& \quad A_27a).(\forall V1k \in (2^{(2^{A_27a})}).(\forall V2s \in (2^{A_27a}). \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1k)) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in \\
& \quad A_27a)\ V0top)\ V2s)))) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a) \\
& \quad V0top)\ (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ V1k))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0S_27 \in (2^{A_27a}).(\forall V1top \in \\
& (ty_2Etopology_2Etopology\ A_27a).((p\ (ap\ (ap\ (c_2Etopology_2Eopen_in \\
& \quad A_27a)\ V1top)\ V0S_27)) \Leftrightarrow ((ap\ (c_2Epred_set_2EBIGUNION\ A_27a) \\
& \quad (ap\ (c_2Epred_set_2EGSPEC\ (2^{A_27a})\ (2^{A_27a}))\ (\lambda V2P \in (2^{A_27a}). \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ 2)\ V2P)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ V1top)\ V2P))\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\
& \quad A_27a)\ V2P)\ V0S_27)))))) = V0S_27))))
\end{aligned}$$