

thm\_2Etoto\_2EStrongOrder\_\_ALT  
 (TMGCJxkQPXzbu-  
 VFH7WSMxciFWwxYxcrhdpD)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
 of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
 of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Erelation\_2Etransitive$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_2F\_5C$

**Definition 7** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 9** We define  $c\_2Erelation\_2Eirreflexive$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_7E$

**Definition 10** We define  $c\_2Erelation\_2EStrongOrder$  to be  $\lambda A\_27g : \iota.\lambda V0Z \in ((2^{A\_27g})^{A\_27g}).(ap (ap c\_2E$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{2}$$

**Theorem 1**

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0Z \in ((2^{A\_27a})^{A\_27a}). ((p (ap (c\_2Erelation\_2EStrongOrder A\_27a) V0Z)) \Leftrightarrow ((p (ap (c\_2Erelation\_2Eirreflexive A\_27a) V0Z)) \wedge (p (ap (c\_2Erelation\_2Etransitive A\_27a) V0Z))))))$$