

thm_2Etoto_2ETO__11
(TMJBgEAHJNy1Mugfw5PMFfjP4tLVBDHEbi7)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (2)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (3)$$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (4)$$

Definition 6 We define $c_2Etoto_2ETotOrd$ to be $\lambda A.\lambda a : \iota.\lambda V0c \in ((ty_2EternaryComparisons_2Eordering$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \quad (5)$$

Let $c_2Etoto_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2Eapto\ A_27a \in (((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}) \quad (6)$$

Let $c_2Etoto_2ETO : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2ETO\ A_27a \in ((ty_2Etoto_2Etoto\ A_27a)^{(ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}}) \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in (ty_2Etoto_2Etoto\ A_27a). ((ap\ (c_2Etoto_2ETO\ A_27a)\ (ap\ (c_2Etoto_2Eapto\ A_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Etoto_2ETotOrd\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Etoto_2Eapto\ A_27a)\ (ap\ (c_2Etoto_2ETO\ A_27a)\ V1r)) = V1r)))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in ((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}). \\ & (\forall V1r_27 \in ((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Etoto_2ETotOrd\ A_27a)\ V0r)) \Rightarrow ((p\ (ap\ (c_2Etoto_2ETotOrd\ A_27a)\ V1r_27)) \Rightarrow ((ap\ (c_2Etoto_2ETO\ A_27a)\ V0r) = (ap\ (c_2Etoto_2ETO\ A_27a)\ V1r_27)) \Leftrightarrow (V0r = V1r_27)))))) \end{aligned}$$