

thm_2Etoto_2ETO__apto__ID
(TMP1aspQqG7DHmi4vzTB2G2ZAwaYHVF74J1)

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Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A \cdot 27a}).(ap (ap (c_2Emin_2E_3D (2^{A \cdot 27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (3)$$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (4)$$

Definition 6 We define $c_2Etoto_2ETotOrd$ to be $\lambda A.\lambda a : \iota.\lambda V0c \in ((ty_2EternaryComparisons_2Eordering$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \quad (5)$$

Let $c_2Etoto_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2Eapto\ A_27a \in (((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}) \quad (6)$$

Let $c_2Etoto_2ETO : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2ETO\ A_27a \in ((ty_2Etoto_2Etoto\ A_27a)^{(ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}}) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0a \in (ty_2Etoto_2Etoto\ A_27a).((ap\ (c_2Etoto_2ETO\ A_27a)\ (ap\ (c_2Etoto_2Eapto\ A_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Etoto_2ETotOrd\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Etoto_2Eapto\ A_27a)\ (ap\ (c_2Etoto_2ETO\ A_27a)\ V1r)) = V1r)))) \end{aligned} \quad (8)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Etoto_2Etoto\ A_27a).((ap\ (c_2Etoto_2ETO\ A_27a)\ (ap\ (c_2Etoto_2Eapto\ A_27a)\ V0a)) = V0a))$$