thm_2Etoto_2ETO__cpn__eqn (TMQTLW9RK2uCn5d1ghGnkCNvNBBWy4UPhD3)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Definition 8 *We define* c_2Etoto_2ETotOrd *to be* $\lambda A\_27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eorde$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{7}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$(((\neg(c\_2EternaryComparisons\_2ELESS = c\_2EternaryComparisons\_2EEQUAL)) \wedge ((\neg(c\_2EternaryComparisons\_2ELESS = c\_2EternaryComparisons\_2EGREATER)) \wedge (\neg(c\_2EternaryComparisons\_2EEQUAL = c\_2EternaryComparisons\_2EGREATER)))) \wedge ((\neg(c\_2EternaryComparisons\_2EEQUAL = c\_2EternaryComparisons\_2ELESS)) \wedge ((\neg(c\_2EternaryComparisons\_2EGREATER = c\_2EternaryComparisons\_2ELESS)) \wedge (\neg(c\_2EternaryComparisons\_2EGREATER = c\_2EternaryComparisons\_2EEQUAL))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V0c)) \Rightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ V0c\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V1x = V2y)))))) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V0c)) \Rightarrow ((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ V0c\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) \Rightarrow (\neg(V1x = V2y))))) \wedge ((\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((ap\ (ap\ V0c\ V3x)\ V4y) = c\_2EternaryComparisons\_2EGREATER) \Rightarrow (\neg(V3x = V4y))))) \wedge (\forall V5x \in A\_27a.(\forall V6y \in A\_27a.(((ap\ (ap\ V0c\ V5x)\ V6y) = c\_2EternaryComparisons\_2EEQUAL) \Rightarrow (V5x = V6y))))))))$$