thm_2Etoto_2ETO__injection (TMKHSnsnyZbR7sSarThVVtKSpg5GbytC9m9)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EONE__ONE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(ap\ (\mathsf{c\_2E}$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Etoto_2ETotOrd *to be* $\lambda A\_27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eorde$

Definition 8 *We define* c_2Etoto_2EimageOrd *to be* $\lambda A\_27a : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27c^{A\_27a}).\lambda V1cp \in (($

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebool\_2EONE\_\_ONE}\ A\_27a\ A\_27b)\ V0f)) \Leftrightarrow (\forall V1x1 \in A\_27a.(\forall V2x2 \in A\_27a.(((ap\ V0f\ V1x1) = (ap\ V0f\ V2x2)) \Rightarrow (V1x1 = V2x2)))))) \tag{8}$$

Theorem 1

$$\forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0cp \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27c})^{A\_27c}).((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27c)\ V0cp)) \Rightarrow (\forall V1f \in (A\_27c^{A\_27d}).((p\ (ap\ (\mathsf{c\_2Ebool\_2EONE\_\_ONE}\ A\_27d\ A\_27c)\ V1f)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27d)\ (ap\ (ap\ (\mathsf{c\_2Etoto\_2EimageOrd}\ A\_27d\ A\_27c)\ V1f)\ V0cp)))))))$$