thm_2Etoto_2ETO__of__greater__ler (TMdMXJe7gLYPLpAoecjH3iy4xDYc3dhHN4a)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Definition 6 *We define* c_2Etoto_2ETotOrd *to be* $\lambda A\_27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eorde$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Etoto_2ETO__of__LinearOrder *to be* $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in$

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 12 *We define* c_2Erelation_2Etrichotomous *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebo

Definition 13 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Erelation_2Eirreflexive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2

Definition 16 *We define* c_2Erelation_2EStrongOrder *to be* $\lambda A\_27g : \iota.\lambda V0Z \in ((2^{A\_27g})^{A\_27g}).(ap\ (ap$ c_2E

Definition 17 *We define* c_2Erelation_2EStrongLinearOrder *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (a$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V0c)) \Rightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ V0c\ V1x)\ V2y) = c\_2EternaryComparisons\_2EGREATER) \Leftrightarrow ((ap\ (ap\ V0c\ V2y)\ V1x) = c\_2EternaryComparisons\_2ELESS)))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2EStrongLinearOrder}\ A\_27a)\ V0r)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ (ap\ (\mathsf{c\_2Etoto\_2ETO\_\_of\_\_LinearOrder}\ A\_27a)\ V0r))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2EStrongLinearOrder}\ A\_27a)\ V0r)) \Rightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Etoto\_2ETO\_\_of\_\_LinearOrder}\ A\_27a)\ V0r)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) \Leftrightarrow (p\ (ap\ (ap\ V0r\ V1x)\ V2y))))))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in ((2^{A\_27a})^{A\_27a}).\ ((p\ (ap\ (\mathsf{c\_2Erelation\_2EStrongLinearOrder}\ A\_27a)\ V0r)) \Rightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Etoto\_2ETO\_\_of\_\_LinearOrder}\ A\_27a)\ V0r)\ V1x)\ V2y) = c\_2EternaryComparisons\_2EGREATER) \Leftrightarrow (p\ (ap\ (ap\ V0r\ V2y)\ V1x)))))))$$