thm_2Etoto_2ETO__onto (TMW8i6CayWnfWDFPkryG3UVxJLnM5ut7Nqp)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Definition 10 *We define* c_2Etoto_2ETotOrd *to be* $\lambda A\_27a:\iota.\lambda V0c\in((ty\_2EternaryComparisons\_2Eord$

Let $ty\_2Etoto\_2Etoto:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{5}$$

Let $c\_2Etoto\_2Eapto:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Etoto\_2Eapto\ A\_27a\in(((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_2} \tag{6}$$

Let $c\_2Etoto\_2ETO:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Etoto\_2ETO\ A\_27a\in((ty\_2Etoto\_2Etoto\ A\_27a)^{((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a})}) \tag{7}$$

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0a\in(ty\_2Etoto\_2Etoto\ A\_27a).((ap\ (c\_2Etoto\_2ETO\ A\_27a)\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0a))=V0a))\wedge(\forall V1r\in((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V1r))\Leftrightarrow((ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ (ap\ (c\_2Etoto\_2ETO\ A\_27a)\ V1r))=V1r)))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0a\in(ty\_2Etoto\_2Etoto\ A\_27a).(\exists V1r\in((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((V0a=(ap\ (c\_2Etoto\_2ETO\ A\_27a)\ V1r))\wedge(p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V1r)))))$$