

thm\_2Etoto\_2ETO\_\_refl  
(TMT12HUGZcFfPLhCyG4jEGDG78qwU1mjUqZ)

October 26, 2020

Let  $ty\_2EternaryComparisons\_2Eordering : \iota$  be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (1)$$

Let  $c\_2EternaryComparisons\_2EEQUAL : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

Let  $c\_2EternaryComparisons\_2ELESS : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \quad (3)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2$

Let  $c\_2EternaryComparisons\_2EGREATER : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \quad (4)$$

**Definition 6** We define  $c\_2Etoto\_2ETotOrd$  to be  $\lambda A.\lambda 27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eordering$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}). \\ ((p\ (ap\ (c\_2Etoto\_2ETotOrd\ A\_27a)\ V0c)) \Rightarrow (\forall V1x \in A\_27a.( \\ \forall V2y \in A\_27a.(((ap\ (ap\ V0c\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow \\ (V1x = V2y)))))) \end{aligned} \tag{5}$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}). \\ ((p\ (ap\ (c\_2Etoto\_2ETotOrd\ A\_27a)\ V0c)) \Rightarrow (\forall V1x \in A\_27a.( \\ (ap\ (ap\ V0c\ V1x)\ V1x) = c\_2EternaryComparisons\_2EEQUAL)))) \end{aligned}$$