

thm_2Etoto_2ETotOrd__TO__of__Strong
(TMZU654jEQF5XMfFdr2Aj4UbWTLmcv897op)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Erelation_2Etransitive$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_27E$

Definition 9 We define $c_2Erelation_2Eantisymmetric$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_27E$

Definition 10 We define $c_2Erelation_2EOrder$ to be $\lambda A_27g : \iota.\lambda V0Z \in ((2^{A_27g})^{A_27g}).(ap (ap c_2Ebool_2E_27E$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 12 We define $c_2Erelation_2Etrichotomous$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_27E$

Definition 13 We define $c_2Erelation_2Eirreflexive$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_27E$

Definition 14 We define $c_2Erelation_2EStrongOrder$ to be $\lambda A_27g : \iota.\lambda V0Z \in ((2^{A_27g})^{A_27g}).(ap (ap c_2Ebool_2E_27E$

Definition 15 We define $c_2Erelation_2EStrongLinearOrder$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (a$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (2)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (3)$$

Definition 16 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge \dots)$ of type $\iota \Rightarrow \iota$).

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V27a.($

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (4)$$

Definition 18 We define $c_2Etoto_2ETO_of_LinearOrder$ to be $\lambda A.\lambda 27a : \iota.\lambda V0r \in ((2^{A.\lambda 27a})^{A.\lambda 27a}).\lambda V1x \in$

Definition 19 We define $c_2Etoto_2ETotOrd$ to be $\lambda A.\lambda 27a : \iota.\lambda V0c \in ((ty_2EternaryComparisons_2Eord$

Definition 20 We define $c_2Erelation_2ELinearOrder$ to be $\lambda A.\lambda 27a : \iota.\lambda V0R \in ((2^{A.\lambda 27a})^{A.\lambda 27a}).(ap (ap\ c_2E$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow (\forall V0R \in ((2^{A.\lambda 27a})^{A.\lambda 27a}). \\ & ((p (ap (c_2Erelation_2EStrongOrder\ A.\lambda 27a)\ V0R)) \Rightarrow (p (ap (c_2Erelation_2EOrder \\ & A.\lambda 27a)\ V0R)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow (\forall V0r \in ((2^{A.\lambda 27a})^{A.\lambda 27a}). \\ & ((p (ap (c_2Erelation_2ELinearOrder\ A.\lambda 27a)\ V0r)) \Rightarrow (p (ap (c_2Etoto_2ETotOrd \\ & A.\lambda 27a)\ (ap (c_2Etoto_2ETO_of_LinearOrder\ A.\lambda 27a)\ V0r)))))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0r \in ((2^{A_{27a}})^{A_{27a}}). \\ & ((p (ap (c_2Erelation_2EStrongLinearOrder A_{27a}) V0r)) \Rightarrow (p (ap \\ & (c_2Etoto_2ETotOrd A_{27a}) (ap (c_2Etoto_2ETO_of_LinearOrder \\ & A_{27a}) V0r)))))) \end{aligned}$$