

thm_2Etoto_2Eapto__inv
(TMGa6ixyE2d19m8QsHUF2PiQzfXLfh1jk9x)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \quad (2)$$

Let $c_2Etoto_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2Eapto\ A_27a \in (((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}) \quad (3)$$

Definition 9 We define $c_2Etoto_2ETO_inv$ to be $\lambda A_27a : \iota.\lambda V0c \in ((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}$

Let $c_2Etoto_2ETO : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2ETO\ A_27a \in ((ty_2Etoto_2Etoto\ A_27a)((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})) \quad (4)$$

Definition 10 We define $c_2Etoto_2Etoto_inv$ to be $\lambda A_27a : \iota.\lambda V0c \in (ty_2Etoto_2Etoto\ A_27a).(ap\ (c_2$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (5)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (6)$$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (7)$$

Definition 11 We define $c_2Etoto_2ETotOrd$ to be $\lambda A_27a : \iota.\lambda V0c \in ((ty_2EternaryComparisons_2Eord$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (13)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (17)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (18)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (23)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.(((\neg(\neg(p V0p))) \Rightarrow (p V0p)))) \quad (25)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty_2Etoto_2Etoto\ A_{.27a}).((ap\ (c_2Etoto_2ETO\ A_{.27a})\ (ap\ (c_2Etoto_2Eapto\ A_{.27a})\ V0a)) = V0a)) \quad (26)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto\ A_{.27a}).(p\ (ap\ (c_2Etoto_2ETotOrd\ A_{.27a})\ (ap\ (c_2Etoto_2Eapto\ A_{.27a})\ V0c)))) \quad (27)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0r \in ((ty_2EternaryComparisons_2Eordering^{A_{.27a}})^{A_{.27a}}).((p\ (ap\ (c_2Etoto_2ETotOrd\ A_{.27a})\ V0r)) \Rightarrow ((ap\ (c_2Etoto_2Eapto\ A_{.27a})\ (ap\ (c_2Etoto_2ETO\ A_{.27a})\ V0r)) = V0r))) \quad (28)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0c \in ((ty_2EternaryComparisons_2Eordering^{A_{.27a}})^{A_{.27a}}).((p\ (ap\ (c_2Etoto_2ETotOrd\ A_{.27a})\ V0c)) \Rightarrow (p\ (ap\ (c_2Etoto_2ETotOrd\ A_{.27a})\ (ap\ (c_2Etoto_2ETO_inv\ A_{.27a})\ V0c)))))) \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0r \in ((ty_2EternaryComparisons_2Eordering^{A_{.27a}})^{A_{.27a}}).((p\ (ap\ (c_2Etoto_2ETotOrd\ A_{.27a})\ V0r)) \Rightarrow ((ap\ (c_2Etoto_2Etoto_inv\ A_{.27a})\ (ap\ (c_2Etoto_2ETO\ A_{.27a})\ V0r)) = (ap\ (c_2Etoto_2ETO\ A_{.27a})\ (ap\ (c_2Etoto_2ETO_inv\ A_{.27a})\ V0r)))))) \quad (30)$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto\ A_{.27a}).((ap\ (c_2Etoto_2Eapto\ A_{.27a})\ (ap\ (c_2Etoto_2Etoto_inv\ A_{.27a})\ V0c)) = (ap\ (c_2Etoto_2ETO_inv\ A_{.27a})\ (ap\ (c_2Etoto_2Eapto\ A_{.27a})\ V0c))))$$