thm_2Etoto_2EcharOrd__eq__lem (TMYUAvGuqQLs2E3hk2sWDzBvsQVnuz1jxnf)

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Let $ty\_2Estring\_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Estring\_2ECHR : \iota$ be given. Assume the following.

$$c\_2Estring\_2ECHR \in (ty\_2Estring\_2Echar^{ty\_2Enum\_2Enum}) \quad (3)$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (4)$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (5)$$

Definition 3 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \quad (6)$$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \quad (7)$$

Definition 6 *We define* c_2Etoto_2ETotOrd *to be* $\lambda A\_27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eorde$

Let $c\_2Estring\_2EORD : \iota$ be given. Assume the following.

$$c\_2Estring\_2EORD \in (ty\_2Enum\_2Enum^{ty\_2Estring\_2Echar}) \quad (8)$$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)$ $(\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap$ $(ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (10)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (11)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge p$
of type $\iota\Rightarrow\iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Etoto_2ETO__of__LinearOrder *to be* $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1x \in$

Definition 15 *We define* c_2Etoto_2EnumOrd *to be* $(ap$ (c_2Etoto_2ETO__of__LinearOrder ty_2Enum_2Enu

Definition 16 *We define* c_2Etoto_2EcharOrd *to be* $\lambda V0a \in ty\_2Estring\_2Echar.\lambda V1b \in ty\_2Estring\_2Ec$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}). ((p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ A\_27a)\ V0c)) \Rightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ V0c\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V1x = V2y)))))) \tag{14}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Etoto\_2ETotOrd}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Etoto\_2EnumOrd})) \tag{15}$$

Theorem 1

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Etoto\_2EnumOrd}\ V0a)\ V1b) = c\_2EternaryComparisons\_2EEQUAL) \Rightarrow ((ap\ (ap\ \mathsf{c\_2Etoto\_2EcharOrd}\ (ap\ c\_2Estring\_2ECHR\ V0a))\ (ap\ c\_2Estring\_2ECHR\ V1b)) = c\_2EternaryComparisons\_2EEQUAL))))$$