thm_2Etoto_2Elistorder__ind (TMaVFKYNX1fBHSr1AGas2FujC9sfiCL3Gbf)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let $c\_2Elist\_2Elist\_\_size : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2Elist\_\_size\ A\_27a \in (\\ (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Enum\_2Enum^{A\_27a})}) \tag{3}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{4}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist \\ A\_27a) \tag{5}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

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Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27\ldots}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P{\Rightarrow}p\ Q)$
of type $\iota.$

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2\ldots$

Let $c\_2Enum\_2EREP\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (8)$$

Let $c\_2Enum\_2ESUC\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \qquad (9)$$

Let $c\_2Enum\_2EABS\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (10)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota{\Rightarrow}\iota.$

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu\ldots$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En\ldots$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in\ldots$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2\ldots$

Let $c\_2Enum\_2EZERO\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \qquad (11)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP).$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\text{c\_2Ebool\_2E}\ldots$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(12)

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(13)

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(14)

Definition 18 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 19 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(15)

Definition 20 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 21 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 22 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 23 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

(16)

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST$$
$$A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

(17)

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

(18)

Let $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

(19)

Definition 24 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 25 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 26 *We define* c_2Erelation_2Einv__image *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27b}).\lambda V$

Definition 27 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ ($c_2Ebool_2E_21

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ \textsf{c\_2Enum\_2E0}) = V0m)) \tag{20}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \textsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ (\\ ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \textsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC} \\ V1n)) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))))) \tag{21}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V1n)\ V0m)))) \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))) \tag{23}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\ \textsf{c\_2Enum\_2E0})\ V0n))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (\neg(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V1n)\ V0m))))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge ($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n)))))))))))$$

$$(26)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ ($$
$$ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$

$$(27)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))$$

$$(28)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))\ V0n)))$$

$$(29)$$

Assume the following.

$$True \qquad\qquad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p$$
$$V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))$$

$$(31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t)))$$

$$(32)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \qquad (33)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge$$
$$(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee$$
$$(p\ V0t))\Leftrightarrow(p\ V0t))))))))$$
$$\tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((\tag{36}$$
$$(p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t))))))))$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge$$
$$((\neg False)\Leftrightarrow True))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow$$
$$(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\tag{39}$$
$$p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in$$
$$A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in$$
$$(2^{A\_27a}).((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\vee(p\ (ap\ V1Q\ V2x))))\Leftrightarrow$$
$$((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\vee(\exists V4x \in A\_27a.(p\ (\tag{41}$$
$$ap\ V1Q\ V4x)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in$$
$$2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x)))\vee(p\ V1Q))\Leftrightarrow(\exists V3x \in \tag{42}$$
$$A\_27a.((p\ (ap\ V0P\ V3x))\vee(p\ V1Q))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ($$
$$2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \quad (43)$$
$$A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in$$
$$2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \quad (44)$$
$$A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ($$
$$(p\ V1B) \vee (p\ V2C)))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg ($$
$$p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B)))))))$$
$$(46)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow$$
$$((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p$$
$$V0t1) \wedge (p\ V1t2)) \vee ((\neg (p\ V0t1)) \wedge (\neg (p\ V1t2)))))))) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}).$$
$$((ap\ (ap\ (c\_2Elist\_2Elist\_\_size\ A\_27a)\ V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) =$$
$$\mathsf{c\_2Enum\_2E0})) \wedge (\forall V1f \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2a0 \in$$
$$A\_27a.(\forall V3a1 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2Elist\_\_size$$
$$A\_27a)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1)) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))) \ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ V1f$$
$$V2a0))\ (ap\ (ap\ (c\_2Elist\_2Elist\_\_size\ A\_27a)\ V1f)\ V3a1)))))))))$$
$$(50)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist$$
$$A\_27a).((V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.($$
$$\exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \quad (51)$$
$$A\_27a)\ V1h)\ V2t)))))$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\textsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \textsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V4n) = \textsf{c\_2Enum\_2E0}))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\textsf{c\_2Enum\_2E0})\ V8n) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \textsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \textsf{c\_2Enum\_2E0}) =$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \textsf{c\_2Enum\_2ESUC}$
$\textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$\textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ \textsf{c\_2Enum\_2E0}) =$
$\textsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \textsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\textsf{c\_2Enum\_2E0} = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n))\Leftrightarrow(V20n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}$
$\textsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\textsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow$$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))$$

$$(53)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))$$

$$(54)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow($$
$$\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a.$$
$$(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)$$
$$V1q)\ V2r)))))$$

$$(55)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow($$
$$\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b)\ V0x)) = V0x))$$

$$(56)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (\mathsf{c\_2Epair\_2EFST}\ A\_27a$$
$$A\_27b)\ (ap\ (ap\ \mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x)))$$
(57)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (\mathsf{c\_2Epair\_2ESND}\ A\_27a$$
$$A\_27b)\ (ap\ (ap\ \mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y)))$$
(58)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in$$
$$A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a$$
$$A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ \mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) =$$
$$(ap\ (ap\ V0f\ V1x)\ V2y)))))$$
(59)

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Eprim\_rec\_2E\_3C}))$$
(60)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$((p\ (ap\ \mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}).$$
$$((\forall V2x \in A\_27a.((\forall V3y \in A\_27a.((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow$$
$$(p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A\_27a.(p\ (ap$$
$$V1P\ V4x)))))))$$
(61)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0R \in ((2^{A\_27b})^{A\_27b}).(\forall V1f \in (A\_27b^{A\_27a}).((($$
$$p\ (ap\ \mathsf{c\_2Erelation\_2EWF}\ A\_27b)\ V0R)) \Rightarrow (p\ (ap\ \mathsf{c\_2Erelation\_2EWF}$$
$$A\_27a)\ (ap\ (ap\ \mathsf{c\_2Erelation\_2Einv\_image}\ A\_27a\ A\_27b)\ V0R)\ V1f))))))$$
(62)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)))$$
(63)

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False)))$$
(64)

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))$$
(65)

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Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\lor(p\ V1B)))\Rightarrow False)\Leftrightarrow \\ ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor((p\ V1q)\lor(p\ V2r)))\land(((p\ V0p)\lor((\neg(\\ p\ V2r))\lor(\neg(p\ V1q)))))\land(((p\ V1q)\lor((\neg(p\ V2r))\lor(\neg(p\ V0p)))))\land((p\ V2r)\lor \\ ((\neg(p\ V1q))\lor(\neg(p\ V0p)))))))))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\land(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor((\neg(p\ V1q))\lor(\neg(p\ V2r))))\land(((p\ V1q)\lor \\ (\neg(p\ V0p)))\land((p\ V2r)\lor(\neg(p\ V0p)))))))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\lor(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(\neg(p\ V1q)))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land \\ ((p\ V1q)\lor((p\ V2r)\lor(\neg(p\ V0p))))))))))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(p\ V1q))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land((\\ \neg(p\ V1q))\lor((p\ V2r)\lor(\neg(p\ V0p))))))))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\lor \\ (p\ V1q))\land((\neg(p\ V1q))\lor(\neg(p\ V0p)))))))) \tag{72}$$

Theorem 1

$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27a})^{A\_27a})}).$
$(((\forall V1V \in ((2^{A\_27a})^{A\_27a}).(\forall V2l \in (ty\_2Elist\_2Elist$
$A\_27a).(p\ (ap\ (ap\ (ap\ V0P\ V1V)\ V2l)\ (c\_2Elist\_2ENIL\ A\_27a)))))\wedge$
$((\forall V3V \in ((2^{A\_27a})^{A\_27a}).(\forall V4s \in A\_27a.(\forall V5m \in$
$(ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (ap\ V0P\ V3V)\ (c\_2Elist\_2ENIL$
$A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4s)\ V5m))))))\wedge(\forall V6V \in$
$((2^{A\_27a})^{A\_27a}).(\forall V7r \in A\_27a.(\forall V8l \in (ty\_2Elist\_2Elist$
$A\_27a).(\forall V9s \in A\_27a.(\forall V10m \in (ty\_2Elist\_2Elist$
$A\_27a).((p\ (ap\ (ap\ (ap\ V0P\ V6V)\ V8l)\ V10m))\Rightarrow(p\ (ap\ (ap\ (ap\ V0P\ V6V)$
$(ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V7r)\ V8l))\ (ap\ (ap\ (c\_2Elist\_2ECONS$
$A\_27a)\ V9s)\ V10m)))))))))))\Rightarrow(\forall V11v \in ((2^{A\_27a})^{A\_27a}).$
$(\forall V12v1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V13v2 \in (ty\_2Elist\_2Elist$
$A\_27a).(p\ (ap\ (ap\ (ap\ V0P\ V11v)\ V12v1)\ V13v2)))))))$