

thm\_2Etoto\_2EtotoELtrans  
(TMUEcSEfrFgjjkVxhK4Ruik9jyWyVMQ5anA)

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Let  $ty\_2EternaryComparisons\_2Eordering : \iota$  be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (1)$$

Let  $c\_2EternaryComparisons\_2ELESS : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.$

Let  $c\_2EternaryComparisons\_2EEQUAL : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (3)$$

Let  $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \quad (4)$$

Let  $c\_2Etoto\_2Eapto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2Eapto\ A\_27a \in (((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a})^{A\_27a}) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto \\ A\_27a). (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. (((ap\ (ap\ (ap\ ( \\ c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow \\ (V1x = V2y)))))) \end{aligned} \quad (8)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto \\ A\_27a). (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. (\forall V3z \in \\ A\_27a. (((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Rightarrow \\ ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow \\ ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \end{aligned}$$