thm_2Etoto_2EtotoLLtrans (TMPGzqSXhioWwKrMfT8uQ3hD46jNawiKngN)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{5}$$

Let $c\_2Etoto\_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2Eapto\ A\_27a \in (((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_} \tag{6}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \Leftrightarrow (V1x = V2y)))) \wedge ((\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V3x)\ V4y) = c\_2EternaryComparisons\_2EGREATER) \Leftrightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V4y)\ V3x) = c\_2EternaryComparisons\_2ELESS)))) \wedge (\forall V5x \in A\_27a.(\forall V6y \in A\_27a.(\forall V7z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V5x)\ V6y) = c\_2EternaryComparisons\_2ELESS) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V6y)\ V7z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V5x)\ V7z) = c\_2EternaryComparisons\_2ELESS)))))))) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS))))))$$