

thm_2Etoto_2Etoto_equal_sym (TMFbqvjB- hYXKdQvV8bEXFKoBQrXcGy5iQeq)

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Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (2)$$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \quad (3)$$

Let $c_2Etoto_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2Eapto\ A_27a \in (((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (V0P))))$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto\ A_27a).(\forall V1x \in A_27a.(\forall V2y \in A_27a.(((ap\ (ap\ (ap\ (c_2Etoto_2Eapto\ A_27a)\ V0c)\ V1x)\ V2y) = c_2EternaryComparisons_2EEQUAL) \Leftrightarrow (V1x = V2y)))))) \end{aligned} \quad (7)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto \\ A_{.27a}).(\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27a}.((ap\ (ap\ (ap\ (\\ c_2Etoto_2Eapto\ A_{.27a})\ V0c)\ V1x)\ V2y) = c_2EternaryComparisons_2EEQUAL) \Leftrightarrow \\ ((ap\ (ap\ (ap\ (c_2Etoto_2Eapto\ A_{.27a})\ V0c)\ V2y)\ V1x) = c_2EternaryComparisons_2EEQUAL)))))) \end{aligned}$$