thm_2Etoto_2Etoto__inv__toto__inv (TMK4pZvS8KDsZvvgsxbu5wXYuLJDTjhF4kM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{1}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2Etoto\_2ETO : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2ETO\ A\_27a \in ((ty\_2Etoto\_2Etoto\ A\_27a)^{((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a})}) \tag{3}$$

Let $c\_2Etoto\_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2Eapto\ A\_27a \in (((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_2} \tag{4}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Etoto_2ETO__inv *to be* $\lambda A\_27a : \iota.\lambda V0c \in ((ty\_2EternaryComparisons\_2Eorde$

Definition 5 *We define* c_2Etoto_2Etoto__inv *to be* $\lambda A\_27a : \iota.\lambda V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(ap\ (c\_2E$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Etoto\_2Etoto\ A\_27a).((ap\ (c\_2Etoto\_2ETO\ A\_27a)\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0a)) = V0a)) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).((ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ (ap\ (\mathsf{c\_2Etoto\_2Etoto\_\_inv}\ A\_27a)\ V0c)) = (ap\ (\mathsf{c\_2Etoto\_2ETO\_\_inv}\ A\_27a)\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_27a}).((ap\ (\mathsf{c\_2Etoto\_2ETO\_\_inv}\ A\_27a)\ (ap\ (\mathsf{c\_2Etoto\_2ETO\_\_inv}\ A\_27a)\ V0c)) = V0c)) \tag{10}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).((ap\ (\mathsf{c\_2Etoto\_2Etoto\_\_inv}\ A\_27a)\ (ap\ (\mathsf{c\_2Etoto\_2Etoto\_\_inv}\ A\_27a)\ V0c)) = V0c))$$