

thm_2Etoto_2Etoto_refl
(TMZo9eBa7tmdSVscS2Gtxnoo8jkRZeGNXQP)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (1)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (2)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (3)$$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (4)$$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \quad (5)$$

Let $c_2Etoto_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etoto_2Eapto\ A_27a \in (((ty_2EternaryComparisons_2Eordering^{A_27a})^{A_27a})^{A_27a}) \quad (6)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto \\ & A_27a).((\forall V1x \in A_27a.(\forall V2y \in A_27a.(((ap (ap (ap \\ & (c_2Etoto_2Eapto A_27a) V0c) V1x) V2y) = c_2EternaryComparisons_2EEQUAL) \Leftrightarrow \\ & (V1x = V2y)))) \wedge ((\forall V3x \in A_27a.(\forall V4y \in A_27a.(((ap \\ & (ap (ap (c_2Etoto_2Eapto A_27a) V0c) V3x) V4y) = c_2EternaryComparisons_2EGREATER) \Leftrightarrow \\ & ((ap (ap (ap (c_2Etoto_2Eapto A_27a) V0c) V4y) V3x) = c_2EternaryComparisons_2ELESS)))) \wedge \\ & ((\forall V5x \in A_27a.(\forall V6y \in A_27a.(\forall V7z \in A_27a.((ap (ap (ap (c_2Etoto_2Eapto A_27a) V0c) V5x) V6y) = c_2EternaryComparisons_2ELESS) \wedge \\ & ((ap (ap (ap (c_2Etoto_2Eapto A_27a) V0c) V6y) V7z) = c_2EternaryComparisons_2ELESS))) \Rightarrow \\ & ((ap (ap (ap (c_2Etoto_2Eapto A_27a) V0c) V5x) V7z) = c_2EternaryComparisons_2ELESS)))))))))) \end{aligned} \tag{10}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0c \in (ty_2Etoto_2Etoto \\ & A_27a).(\forall V1x \in A_27a.(((ap (ap (ap (c_2Etoto_2Eapto A_27a) \\ & V0c) V1x) V1x) = c_2EternaryComparisons_2EEQUAL)))) \end{aligned}$$